N O T E S   O N

HYDRAULIC MEASUREMENTS,

PREPARED FOR THE USE OF

STUDENTS IN CIVIL AND SANITARY ENGINEERING

A T   T H E

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HYDRAULIC MEASUREMENTS.

1. Introductory. Measurements of the volume of flowing water are frequently required in investigations of the amount obtainable from streams for the public supplies of cities, for power development, for irrigation, or for navigation canals; in its distribution from pipes or canals among the different users; in planning river improvements; in tests of hydraulic machinery; and for many other purposes. The mode of measurement to be employed is determined by the quantity of water to be dealt with; by the nature of the channel, which may be a pipe, aqueduct, sewer, canal, or natural stream; by the degree of accuracy required; by the expenditure of time or money involved; and by other circumstances peculiar to each case.

A very small flow may sometimes be conveniently received and measured in vessels of known capacity; or in vessels arranged upon scales, thus permitting the volume to be found from the weight.

For medium quantities resort must usually be had to less direct methods involving the use of formulae with experimentally-determined coefficients, as in the case of the standard orifice, weir, Venturi meter, nozzle, or Pitot tube, which devices, with the accompanying theories, will be studied in theoretical Hydraulics.

Large volumes flowing in open channels generally require the use of floats or current meters, and it is mainly to measurements by such means that the following notes will be confined.

2. Relation between Discharge, Mean Velocity and Area. The purpose in gaging a stream is to determine with reasonable approximation its discharge, that is to say, the volume passing a given cross-section in a unit of time.

If Q = discharge, in cubic feet per second, often called "second-feet",
A = area of cross section, in square feet,
V = mean velocity normal to cross-section, in feet per second,
then $Q = AV$.

The area of cross-section can be determined from soundings; but the mean velocity for the cross-section as a whole can seldom be directly observed with satisfactory accuracy and, consequently, a closer approximation is usually sought by conceiving the section subdivided in a regular way into partial areas, most often vertical strips, for each of which in turn its mean velocity can be found with tolerable closeness.

If a = a partial area, in square feet,
v = the mean velocity past that area, in feet per second,
then av = the corresponding partial discharge,
$Q = \Sigma av$ = the entire discharge,
and finally $V = \frac{Q}{A}$.

3. Choice of Location for Discharge Measurements. The place of measurement may be closely fixed by special requirements, but if not, a wise choice of site will have much to do with the accuracy of the results, and with the convenience and cost of securing them. A proper selection is therefore very important.

It is desirable that the cross-section have a fairly smooth and regular outline, and that it be located upon a straight reach, the longer the better, of tolerably constant width and depth. Such conditions favor the accurate determination of the area; promote freedom from the cross currents and eddies which result from curves, irregular banks, bowlders, ledges, snags, piers, and other obstructions; and tend to ensure that the velocities observed be normal to the cross-section.

In a large proportion of measurements the mean velocity past each of a series of verticals in the cross-section is obtained by observing the velocity past one point in each vertical, or perhaps two or three points, and assuming that the mean for the vertical can be derived from such observations by assuming certain common relations to hold between them; but, in order that it shall be safe to assume such relations, it is essential that the site possess the regular features above mentioned.

If a river station is to be a permanent one, gagings will be made at various stages from low to high water, and the relation thus determined between discharge and gage height; but, in order that this relation shall be constant, it is necessary that the bed and banks be stable, and that the section be free from irregular back water, such as might result if it were within the influence of varying draughts from a mill-pond below, or of temporary obstructions such as ice or log jams, or of tides, or of set-back from a stream in flood entering below the site. A location above rapids gives an approach to the favorable conditions above a measuring weir and is advantageous; as also is one below any lake or extensive stretch of slack water that acts as an equalizer of the stream flow.

Since, at a permanent station gagings will be required both at high and at low water, it is important that the site be sufficiently favorable to accurate measurements in both stages. It might be suitable for high-water gagings, but very poor for those at low stage, on account of sluggish current or partial dead water at the latter time. It is therefore safer to make a selection in low water than in high, especially since information as to discharge in low and medium stages is generally more useful than that for high stages.

It is also desirable that at a permanent station the banks be not subject to overflow in high water, or that, if overflowed, they at least be free from trees or undergrowth. At velocities much below one-half foot per second measurements by current meter are apt to be unreliable, and the U. S. Geological Survey does not accept a location for a permanent station where the

velocity is below this limit in more than 15 per cent. of the cross-section.

Ease and safety in making observations are to be considered. Gagings from bridges supported on piers are said to be as a rule less accurate than those in clear channels; nevertheless, certain types of bridges are very convenient for operations, and because of this and the safe support may well be chosen if the piers are parallel to the current. Measurements by wading may be practicable in low stages, and from an anchored boat, or a boat secured to cross lines in higher water. But when neither wading, nor use of a boat nor of a bridge is possible, resort may still be had to a permanent cable stretched over the stream, with a suspended movable car for the observer, or in a very large river to the use of a heavy power-boat.

The cross-section or sections to be used in gagings should be closely normal to the general direction of flow, which may be judged roughly from the trend of the banks, but more accurately by observing the courses of a number of rod or sub-surface floats.

4. Gage. It is always well to be able to state definitely at what stage of water a gaging was made, and usually a reading of the stage at such time is absolutely necessary. It is obviously so in the case of a permanent station where the relation between gage height and discharge is to be established, and the daily discharge thereafter to be inferred from the daily gage readings. It may also be necessary for the following reasons: The cross-sectional area, A, to be used in the formula, $Q = A V$, must have a definite value corresponding to the average water level prevailing during the taking of observations for V, which observations may or may not be made at the same time as the soundings from which A is to be found. If made at a different time and stage, adjustment of the resulting area to that required in the computation for Q can evidently be made only from a knowledge of the respective positions of the water surface, as given by gage readings. Again, during the taking of soundings in a wide stream the water level may materially change, and the soundings can be adjusted to a common level only by means of the corresponding gage readings. Finally, the area determined from soundings made at a particular stage, supplemented if necessary by levels carried up the banks, may afterward be adjusted to any other stage, without the taking of new soundings, if corresponding gage readings have been made. The change in area is simply that of a horizontal strip, of height equal to the difference in gage readings.

A gage should, therefore, be established at or near the place of measurements, and read at suitable intervals during the progress of either soundings or velocity observations, to the nearest tenth or hundredth of a foot, as may seem justifiable. In the case of the largest rivers two gages are needed, one at each shore, as a cross wind may produce noticeable difference of level within the width of the stream.

To permit precision in readings when the water surface is roughened by wind or otherwise, the gage may be enclosed in a "stilling box" within which the water surface will be smooth. In gagings on the Niagara river at Buffalo a box 7 inches square and 7 feet long, with closed bottom and three ¼-inch holes on the river side, was used, the gage itself being a graduated rod supported by a float within the box. Similar in principle is the use of a small side basin close to the stream, with a pipe or other connection between them. The surface in such a basin may be further stilled, if necessary, by a floating piece of plank, or even by pouring on oil. To effectively prevent wave action within such enclosures Hoyt and Grover state that the aggregate area of openings should not exceed one-half of 1% of the cross-sectional area of the basin (River Discharge, p.30). It is believed that the level obtained by the above devices is a correct mean between wave crest and wave trough.

The gage should be so graduated and set as to avoid the inconvenience of minus readings, which may be accomplished by arranging that the zero of the gage shall be several feet below the lowest known low water, or preferably, in permanent channels, level with the deepest part of the cross-section.

The following styles of gage are to be noticed:-

(a) Vertical scale-board, rigidly attached to tree, pile, bridge pier, or other stable object.

(b) Inclined scale-board, firmly secured and conforming to slope of banks, and graduated so that its figures shall correspond to the usual vertical intervals. Sometimes used where a fixed vertical scale is impracticable (Rept. Chf. of Engrs., U.S.A., 1891, pp. 3820-1).

(c) Weight lowered by chain from a bridge to contact with water surface when reading is desired (see above reference).

At important stations, usually for moderate fluctuations of level, a continuous record of stage is sometimes maintained by means of a self-registering gage or fluviograph, the main features of which are a simple float, or a float plunger compressing air within a manometer; registering mechanism, with revolving drum and clock; and the device for conveying the motion of the float to the register. On paper wrapped around the drum a record is traced both of time and of corresponding water level. (U.S.G.S. Water Supply and Irrigation Paper No. 95, p. 25; Annales des Ponts et Chaussees, 1886, 2d semestre, p. 706).

5. Soundings. It is necessary to determine the outline of the cross-section in which velocities are measured in suf-

ficient detail so that when, in computing the discharge, the section is divided into imaginary vertical strips, the area of each strip can be calculated with proper accuracy. For this purpose soundings are taken. The distances between these, and the closeness with which depths are recorded, should be consistent with the evenness and firmness of the river bed, and with the general accuracy sought, or probably attainable, in the gaging as a whole. It is well to make the intervals equal, except where intermediate soundings may seem necessary to show important irregularities in the bed. A safe principle to follow is to seek so to space the soundings that undue error shall not result from assuming the profile a straight line from one to another. The general rule adopted by the U. S. Geological Survey is to take soundings as follows:-

For streams from	5 to 10	ft. wide, at intervals of	1 ft.
"	10 to 30	" "	2 to 5 ft.
"	30 to 100	" "	5 to 10 "
"	over 100	" "	10 to 25 "

For the largest rivers the intervals should doubtless be still greater.

Depths will ordinarily be read, directly or by estimate, to tenths or half-tenths of a foot; but in artificial channels with smooth linings readings to hundredths may be appropriate. In the violent currents often accompanying floods soundings may be difficult or impracticable, and better results obtained, if bed and banks are stable, by correcting soundings observed in low stages. Current meter gagings require but a single cross-section to be used, and it is a common practice to take soundings in each meter vertical, with intermediate ones if needed, the meter weight and supporting cord often serving as sounding apparatus.

The following types of equipment for sounding are employed:-

(a) Graduated rod, with flat shoe for soft bottoms. Suitable for moderate depths.

(b) For considerable depths, a lead or iron weight, ranging from 5 to 40 lbs., according to depth and swiftness of current, with line. Flat weight advantageous for soft bottom, but longer cylinder otherwise best and should have conical cavity in bottom, to be filled with tallow if samples of bed are desired. For heavy weights a reel is useful, and depth may then be found by counting number of turns of reel in raising lead (See Rept. Chf. of Engrs., U.S.A., 1892, p.3119).

For the line is variously used:-

(1) Italian hemp, 1/4" to 3/8" diameter for heavy weights, well stretched, tagged, and regularly tested before and after use. Sea grass,

braided cotton, etc., also in use. Testing of line very important; corrections as large as 12 feet in 70 feet have been found necessary in lines used in important work.

(2) Piano or other wire, metallic ribbon, or chain, graduations indicated by brazed marks, tags, or otherwise.

In gagings with floats, the value of V in the formula, Q = A V, is presumed to be the mean for the length of the measured course over which the floats run, ranging from 200 or 300 feet downward, according to size and velocity of stream, and clearly A must be the mean area for the same distance. In this case it is therefore necessary to take soundings in detail not only upon the upper and lower cross-sections, but also upon sufficient intermediate ones to determine the mean area with appropriate accuracy.

Soundings must be located when taken so that their position may be known and, if necessary, plotted upon the profile of the cross-section. The following methods of location are applicable, according to circumstances:-

(a) Distance from bank, or from reference line on bank, read on chain, tape, or graduated line, stretched across the stream. Suited to channels of moderate width.

(b) Distances read by stadia from shore end of range.

(c) Angles from base line on shore to sounding stations measured with transit.

(d) Sextant or goniograph angles to known points on shore observed from boat.

(e) Boat brought to intersection of range lines previously established on shore. Common on very large rivers where it is desired to take repeated observations at given points. (See Rept. Chf. of Engrs., U.S.A., 1891, p.3532; also Johnson's "Surveying", p.289).

When the stream surface is not affected by a cross wind there is probably no material error, at a properly located gaging station on any river, in assuming the water line of the cross-section to be level. Even when the surface is smooth, however, there is evidence that the line is not in all cases strictly level. In gagings below the Vyrnwy reservoir, the surface at the center of the stream (width 40') was found slightly lower than at the sides in certain stages. During gagings of the Niagara river at Buffalo (width 1800') the water surface at the center was about 0.2 foot lower than at the side. On the lower Mississippi river the reversal of the current from

one bank to the other at bends causes a difference of level between opposite banks sometimes as great as a foot, and gives the river in the vicinity a warped surface. Some observations on large streams have seemed to show a convex surface during a rising stage, plane during steady flow, and concave during a falling stage, but evidence as to this is not conclusive.

6. Interpolation of Soundings. If soundings have been taken at irregular intervals in the cross-section, and depths be desired at regular intervals for purposes of computation or otherwise, these may be obtained by direct interpolation upon the profile, assuming it either straight between observed soundings, or curved, as may be thought best.

It sometimes happens in the case of a river with shifting bed that soundings are repeated at the same points after an interval of some days, or even weeks, and that at an intermediate date velocities are observed for the purpose of ascertaining the discharge. The cross-section must be estimated as well as possible for the time when velocities were measured. If the conditions are such that the scour, or fill, in the bed may fairly be assumed to have progressed uniformly between the successive dates of sounding, and gage heights have been read, the observed soundings may be corrected to their probable value at the intermediate date as follows:-

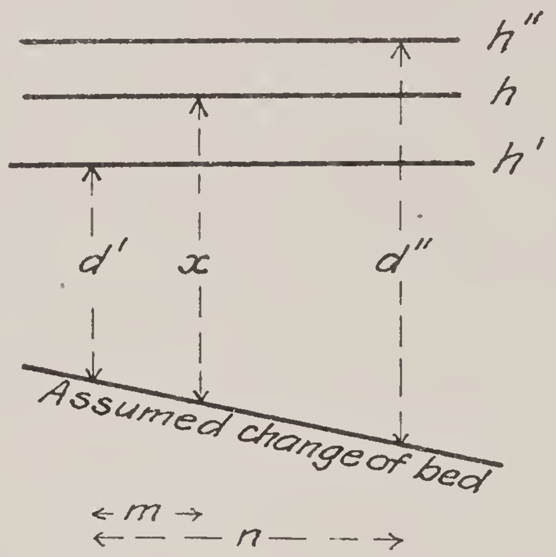


Fig. 1

Sketch a figure on which depths and time intervals are conceived as laid off to scale (Fig. 1).

Let d' and d" = depths as observed at the same station on two different dates.
x = required depth at intermediate date.
h', h" and h = corresponding gage heights.
m = time interval between h' and h.
n = time interval between h' and h".

Then x may easily be found from the geometrical relations of the figure.

7. Computation of Area from Soundings in a Single Cross-Section. As a rule, in computing discharge the area of the cross-section is needed in detail for each of successive vertical strips into which it is conceived to be divided. Usually the strips are of equal width, and either lie between successive observed soundings or have such soundings at their centers. On any plotted cross-section, however, strips may be arbitrarily laid out independently of observed soundings, if there is any call for doing so, and the accompanying depths scaled or numerically interpolated between soundings. Generally the bed

profile is assumed as a series of straight lines extending from one sounding to the next; sometimes, and more accurately, as a series of parabolic arcs, convex downward. Perhaps most often the strips are combined singly with their corresponding velocities in computing the discharge; while at other times, to shorten somewhat the labor of computation, double strips are taken. The problem then is to find with reasonable approximation the areas of the strips.

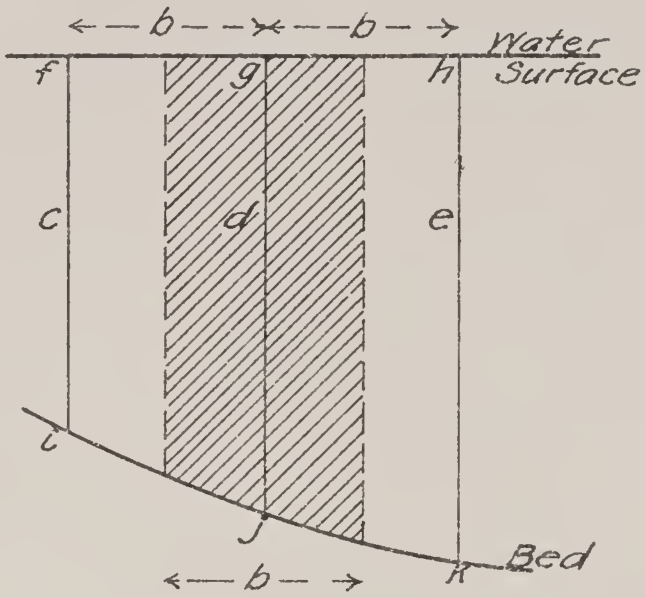


Fig. 2

If a = area of a strip in square feet,
b = its width in feet,
x = its mean depth in feet,
then a = bx.

In Fig. 2 let c, d, e represent successive observed or scaled depths, equally spaced.

(1) Area of any strip may be measured by planimeter.

(2) For any strips, as f g j i, regarded as a trapezoid, $a = b\,\frac{c + d}{2}$

(3) For cross-hatched strip, regarding d as mean depth,

$$a = bd$$

(4) For cross-hatched strip, regarding i j and j k as straight lines,

$$a = b\,\frac{c + 6d + e}{8}$$

which may also be written

$$a = b\,\frac{d + (c-d) + (e-d)}{8}$$

(5) For double strip, regarding each single strip as a trapezoid,

$$a = 2b\,\frac{c + 2d + e}{4}$$

(6) For double strip, more accurately, regarding i j k as a parabolic arc,

$$a = 2b\,\frac{c + 4d + e}{6}$$

At each end of a cross-section there is frequently a strip of odd width or shape, the area of which may be estimated by whatever method commends itself to the judgment.

8. Computation of Mean Area from Soundings in Several Cross-Sections. As noticed in Art. 5, it is necessary in the case of gagings with floats to find the mean cross-section for

the distance covered by the float runs. In Fig 3 let BB and B'B' represent the water's edge; C N a base line on shore; C C' and N N', at right angles to the base line, cross ranges fixing the length of run; and D D' etc. intermediate equally-spaced cross ranges, if such be required for the proper determination of the mean section. The number of these will depend upon the regularity of the bed, the length of the course, and the general accuracy to be expected in the gaging as a whole. Soundings are taken at the same distances out from the base on each cross range, or if not may be interpolated at such distances.

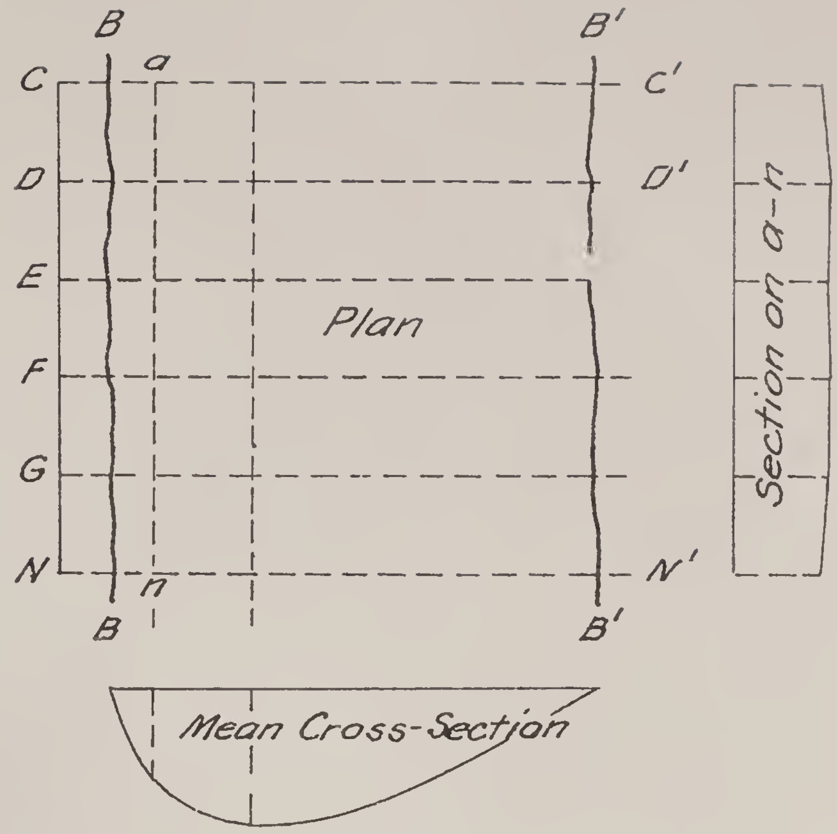


Fig. 3

The mean cross-section may now be constructed, as shown, from a series of ordinates corresponding to the mean depths along successive longitudinal sections, such as a-n. The mean depth of any such longitudinal section, shown at the right, may be found from the numerical mean of depths at the intersected cross ranges or, if greater accuracy be required, by computing the area of the section and dividing by its length. The mean distance out from base line to water's edge may be similarly found, and the representative cross-section is then complete.

9. Complex Motion of Flowing Water. In a general sense, and approximately, the flow in a river or other open channel may be "steady", and it may be allowable, as a matter of convenience, to speak of particles of water moving in "stream lines", and of "threads" of velocity; but careful observers are agreed that, except perhaps when the velocity is exceedingly low, the actual motion of particles, even when the stream appears to the eye entirely tranquil, is extremely unsteady and complex. The velocity is not really constant at any point, but everywhere varies from instant to instant. Innumerable whirlpools are formed, and solid particles in suspension have a rotary motion, both horizontally and vertically, which interferes materially with their settling to the bottom. The variability in direction is shown by this confused motion of particles of silt, by the irregular paths often taken by floating chips, by the swaying of weeds, etc. The paths of individual particles are evidently interlaced irregularly from one moment to another in the most intricate manner.

The "thread" of maximum velocity is never long in the same place, but constantly sways from side to side and rises and falls.

In the Mississippi river at Arkansas City it has been known to move laterally 1800 feet in one day, and half that distance between morning and afternoon. Leviavsky, studying with floats the currents near the surface in the river Dnieper, near Kieff, found no portion of the river where all motions were parallel; there were points on the water surface toward which floating bodies tended from both sides, and others at which if two floats were started close together they would invariably separate within a certain distance (Eng. News, Sept. 1, '04, p.184). It is plain, then, because of the uncertain motions, that velocities must be measured at numerous points in the cross-section, and that even then the difficulties may be considerable in the way of securing a close approximation to the average velocity past the section at any given time.

10. Velocity Pulsations. Even while the wetted cross-section and discharge of a stream, and consequently its mean velocity, remain substantially unchanged, i.e., while the flow remains technically "steady", it is a matter of common observation that the forward velocity at any particular point is subject to constant pulsations, often of considerable magnitude. These pulsations, which sometimes mean a variation of velocity amounting to 50 per cent. of the greater value within one minute, become perceptible to the eye in the behavior of floats, and to the ear in the varying intervals between the taps or buzzes indicating revolutions of the electrical current meter. Cunningham found in the Ganges canal a number of similar floats, run in rapid succession over the same course, to show commonly a range in velocity of as much as 20 per cent. of the mean. Francis observed tube floats, run in a canal at Lowell, under apparently identical conditions, to vary in velocity from about 8 ½ per cent. above the mean to about 11½ per cent. below it. Marr's measurements in the Mississippi at Burlington showed the velocity at 9 feet depth, as given by individual one-minute current meter runs, to vary from the mean for 32 minutes by from about 10 per cent. above to about 11 per cent. below.

These pulsations are observed to occur in streams of all sizes, and are evidence of an irregular motion in the water which has been likened to the unsteady motion of the wind shown in the swaying of weather vanes and the fluttering of pennons. The cause of the unsteadiness is not well known. The surface level of a stream is said also to show usually a pulsation covering from 30 to 50 or more seconds, and giving a vertical oscillation through several hundredths of an inch (U.S.G.S. Water Supply and Irrigation Papers, No. 56, p.42); but Henry's experiments in the St. Clair and Niagara rivers indicated that the velocity pulsations were not synchronous with oscillations of the surface level, which itself varied there very irregularly. Francis attributed the varying velocity of tube floats, as noticed above, to the constant interchange of place of currents of different speed.

Experiments by Marr in the Mississippi, with simultaneous observations on several meters, showed the changes in velocity

to occur at about the same time from surface to bed, but to increase in magnitude as the latter was approached. This latter law has frequently been noted by others. Harlacher observed velocities in the Rhine to vary in a few seconds 20 per cent. at the surface, and 50 per cent. at the bed. Unwin found in the Thames the velocity as measured in individual meter runs of 100 revolutions each to vary from the mean given by a continuous run of 1600 revolutions by from

+ 8.3% to - 6.0% at one-half metre depth, and
+ 16.1% to - 37.4% at six metres depth.

Upon plotting times as abscissas and velocities as ordinates it has been shown that curves result displaying two sets of waves,- minor ones of from 15 to 60 seconds amplitude, and major ones of from 3 to 6 minutes, or even longer. Unwin found for the Thames that, plotting velocities as obtained from successive runs of 100 revolutions each he obtained a very irregular curve, but from successive runs of 500 revolutions each very nearly a straight line, the meter making 500 revolutions during each of a dozen successive equal periods of time (Min. Proc. Inst. Civ. Engrs., Vol.71, p.41).

From what has been said it must be seen that the velocity pulsations have an important bearing not only upon the selection of an instrument for measuring velocity, but also upon the proper length of observations. A float, for example, may move down stream under the influence of a maximum or a minimum impulse, and only by chance with the true average velocity, which therefore can be learned only by taking the mean given by many repetitions. In Major Cunningham's Roorkee hydraulic experiments on the Ganges canal it was thought to be demonstrated that about 50 repetitions with floats were there necessary to obtain a fair average. On the other hand, with a current meter, held at one point, the fluctuations may be averaged simply by increasing the duration of the run.

Baum concluded from his studies in the Rhine that for accurately averaging the pulsations at a point it was necessary to take continuous observations for as long as an hour. Generally, however, it is considered that the longer waves of velocity may be covered and a satisfactory average be obtained by continuing a meter run at a given point for from 5 to 10 minutes, and this has been a common practice with government engineers in this country. Studies by the Hydrographic Department of Hungary, in the river Theiss, have indicated that three or four maxima and minima may be expected in a period of about 3 minutes where the maximum velocity is as high as 2 metres per second, and in a period of about 5 minutes in the slow filaments near the bed, and the department engineers have therefore adopted the rule of making the duration of measurements from 3 to 5 minutes, according to the speed of current (Annales des Ponts et Chaussees, 1898-3, p.301).

Since any single observation with a float, or any short observation with a meter, clearly gives but an accidental value

for velocity, which may even be a maximum or a minimum, it is plain that for the purpose of critical comparison of velocities at different points only many repetitions with floats or long-continued runs with meter can be of much value. To apply such a requirement, however, at the many points which it might be desirable to occupy in the ordinary gaging of a stream of large cross-section would call for an expensive outlay of time, and would involve the further serious difficulty that before the observations had progressed far a change in stage, and consequently in the amount and distribution of velocity throughout the cross-section, might occur. A compromise has, therefore, usually to be accepted between duration and number of observations, and meter runs of 5 or 10 minutes each are made at a moderate number of points, or runs of say 1 minute, more or less, are made at a larger number. The shorter the run, the greater the necessity of increasing the number of observations and of distributing them well over the discharge section. With floats, it is common to make single runs at numerous stations well distributed across the stream, and then to draw an averaging curve through the velocity points thus determined.

11. <u>General Law of Variation of Velocities past a Cross-Section</u>. In spite of the intricacy and variability of the paths of the individual particles of water, and the pulsations in velocity at any given point, there is, for the mass of flowing water as a whole, a general law of distribution of velocities in the cross-section, upon a knowledge of which are based a large proportion of stream gagings. On account of the roughness of the channel lining, giving rise to numberless eddies, and the viscosity of the water, a resistance to the flow is developed next the bed and banks and is felt, in lessening degree, as these are receded from; so that in a general way the highest velocities are found in the central part of the stream, and well toward the surface. It is noticeable, however, that generally the maximum of the velocities past the different points of any vertical line is found, not <u>at</u> the water surface, as might be expected, but <u>below</u> it, and sometimes as far down as mid-depth.

The cause of this depression has been a matter of much speculation, and as long ago as 1867 Francis made experiments in the Northern and Western canals at Lowell, which were thought to show that the relatively slow movement at the surface is due to eddying masses of slow-moving water near the bed being detached and forced upward to the surface by quicker currents; and others have supported substantially the same view. Cunningham, however, considered the air to supply an efficient resisting margin, and the surface velocity to be necessarily thereby retarded. No doubt a strong wind with or against the current affects the position of the thread of maximum velocity, but it appears that even with a wind blowing down stream the maximum velocity may often be found below the surface, and air resistance seems an inadequate explanation of the phenomenon in question.

The theory best agreeing with observed facts appears to

be that elaborated by Stearns (Trans. Am. Soc. C.E., Vol. 12, pp. 331 et seq.), according to which there is an upward flow at the sides of a channel, carrying with it the slow-moving water always found in the immediate vicinity of channel linings, which upon reaching the surface flows obliquely toward the middle of the stream, retarding by its slower movement the velocity of the surface layers and constituting thereby the main cause of the depression of the thread of maximum velocity below the surface. This upward and outward flow was made evident by placing a vertical projecting board at the side of a rectangular canal, when sawdust that had been mixed with the water was seen to rise along the up-stream surface of the board to the water surface and there to spread out toward the center of the channel. Cunningham thought such outward flow well demonstrated by the behavior of surface floats in experiments in the Ganges canal, it sometimes having been found necessary to make 100 runs before 3 could be obtained that followed a fair course only a dozen feet long and about 8 inches from a straight vertical bank, the floats showing a constant tendency to swerve outward.

Inasmuch as the locus of the thread of mean velocity for any vertical line is likely to rise and fall with that of the thread of maximum velocity, it is of much practical importance to notice the effects upon the latter which should follow the application of Stearns' theory, and which seem to be generally realized in practice. Other things equal, the locus of the thread of maximum velocity, in any vertical line -

(a) Should be higher, the smoother the channel lining, and vice versa.

(b) Should be higher, the shallower the water section, and vice versa; and for a similar reason it should be higher, the more gently the bed slopes away from the water's edge, and vice versa. In the central part, at least, of relatively shallow streams the maximum velocity is apt to be found at or very near the surface.

(c) Should be higher at the greater distance from the sides, and vice versa. In the La Plata river, several miles from shore, Revy found the maximum velocity at the surface, though the water was there 25 feet deep.

Fig. 4, showing the results of observations by Darcy, makes clear to the eye the general distribution of velocities to be expected in a rectangular section, and displays not only curves connecting the loci of equal velocities in the cross-section,

but also curves of variation of velocity in selected vertical lines, and others similarly in horizontal lines; they illustrate well some of the laws stated above.

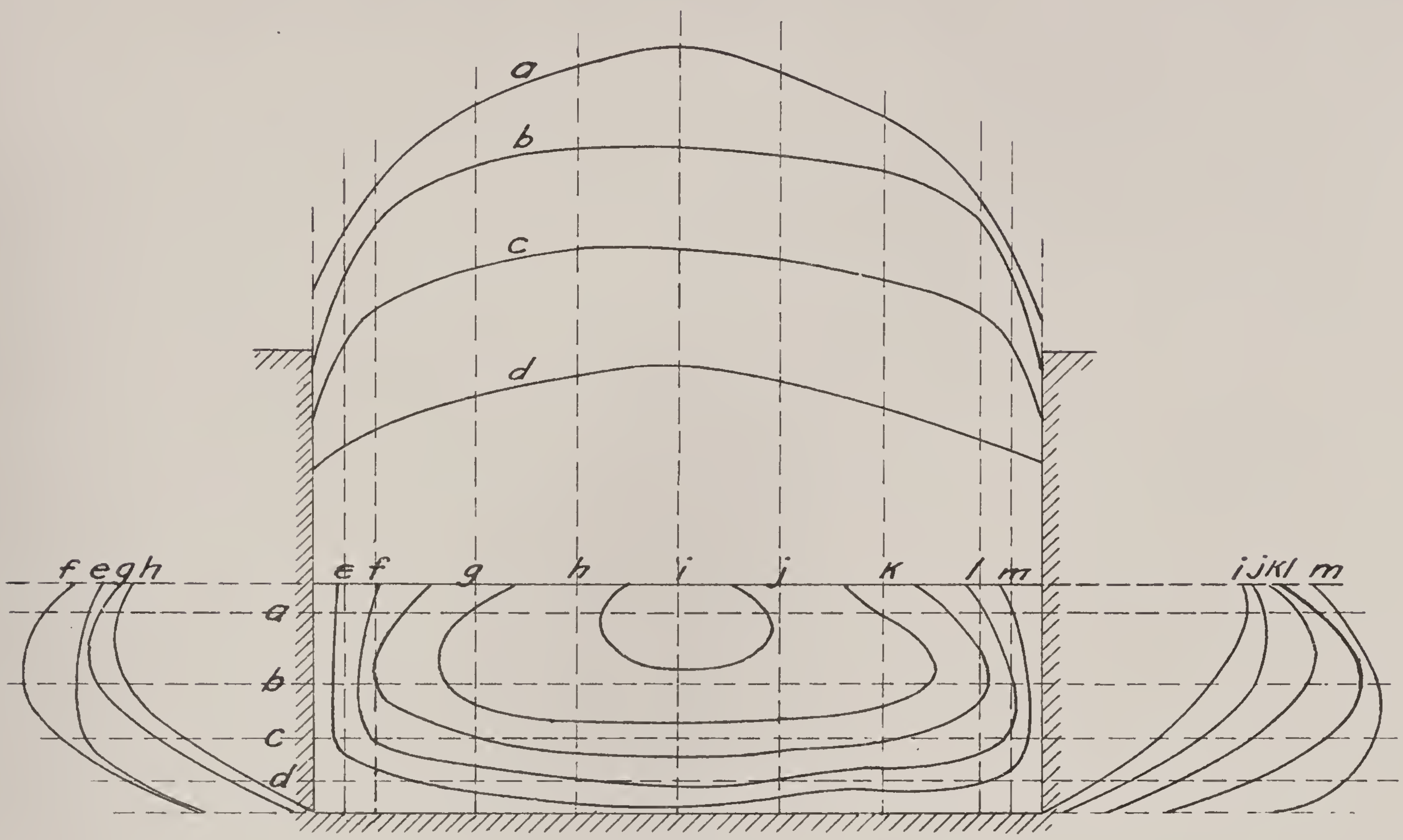


Fig. 4

12. Velocity Measurements at Numerous Points Uniformly Distributed over the Cross-Section. Since it is impossible to measure all the different velocities in a cross-section, and since a reasonably close approximation to the mean velocity and discharge is all that is required, it is apparent that if the cross-section can be satisfactorily subdivided into imaginary equal elementary areas, and the velocity observed at the center of each of these, the numerical mean of the observed velocities will be, approximately, the desired mean velocity for the entire section.

If a = the common elementary area,
n = the number of such areas,
A = the entire cross-sectional area,
v = the observed velocity at center of each elementary area,
V = the mean velocity for the entire cross-section,
and Q = the total discharge;

then $V = \frac{\Sigma v}{n}$,

and $Q = A\,V = \Sigma\, av$.

This method can successfully be applied to artificial sections

of simple geometrical shape, in which the flow is sufficiently steady not to change materially during the time required for the "point measurements", and a high degree of accuracy may be attained, say within 1% of the truth (e.g. see Watkins gaging in old Croton aqueduct, mentioned by Freeman in Report on New York's Water Supply, p.155)

13. Reduction Coefficients. On account of the great time and expense that would be necessary for gagings in large channels if based simply upon a wide distribution of point measurements, and because of the danger of a total change in the conditions of flow during a lengthy operation, very extensive studies have been conducted in the effort to discover such relations between the various velocities in the cross-section as would permit of reducing to a minimum the necessary number of observation points; these being so chosen as either to give directly certain desired mean velocities, or to give them through the application of "reduction coefficients" to the observed values. (In the Roorkee hydraulic experiments about 50,000 velocity measurements were made with this general aim in view). Such relations have been sufficiently well determined to ensure under favorable conditions a fair approximation to the discharge, and yet it is true that they are so affected by differences of width, depth, velocity, or channel lining, that as the number of observation points is diminished the likelihood of close accuracy in the computed result is correspondingly lessened.

14. Velocity Measurements at but One Point in the Cross-Section. There appears to be no point in the cross-section the velocity at which bears, under varying conditions, a sufficiently constant relation to the mean velocity for the entire section to ensure the determination of the latter velocity with closeness from observations at the single point. Nevertheless, by measuring either the central surface velocity, in artificial channels, or the maximum surface velocity, in natural channels, and applying a mean reduction coefficient of say 0.8, a rough approximation to the discharge can be made, with an error perhaps not likely to exceed 10%; and by experimentally determining a special coefficient for the particular cross-section used subsequent measurements could doubtless be made with much greater accuracy. This method may be in order in a reconnoissance, where great accuracy is not essential; or in hasty measurements of flood discharge, where, also, close accuracy is often not needed.

If Q = discharge,
A = entire cross-section,
V = mean velocity past entire cross-section,
V_s = observed surface velocity,
C = reduction coefficient, = $\dfrac{V}{V_s}$,

then $Q = A\,C\,V_s$.

Measurement of the central surface velocity was regularly

practised in the early history of gagings in certain water-power canals at Lowell, Mass., by finding the velocity of small surface floats at mid-stream, and using reduction coefficients which were independently determined at 0.847 for the Western canal (width about 27 feet, depth about 8 feet, mean velocity about 2.7 feet per second); and at 0.814 for the Merrimack canal (width about 30 feet, depth about 8 1/2 feet, mean velocity about 1.7 feet per second); but the uncertainty as to whether reliable results could be obtained by the same method for other canals of the system presenting less favorable conditions led to its entire abandonment.

The numerical value of this coefficient appears to range, under different conditions, between 0.75 and 0.95. In extensive gagings below the Vyrnwy reservoir (stream bed 32 feet wide, bank slopes 2 base to 1 vertical) it was found to range between 0.78 and 0.94, with an average of 0.83. It seemed there to be independent of depth and velocity, but a study by Prony of experiments by Du Buat in small wooden troughs led him to the following formula in which the coefficient is seen to increase with the velocity:-

$$C = \frac{V}{V_{cs}} = \frac{V_{cs} + 7.78}{V_{cs} + 10.35}$$ (Lowell Hydraulic Experiments, p.154),

V_{cs} representing the central surface velocity.

For observed surface velocities ranging from 1 to 10 feet per second this formula will be seen to give values of the coefficient ranging from 0.77 to 0.87.

It is urged, however, that the surface velocity at any particular point, such as mid-stream, is easily affected by wind and is otherwise very variable, and that better results will be obtained by observing the <u>maximum</u> surface velocity, which may or may not occur at mid stream; in the Vyrnwy experiments its position was found not to be constant, but to be generally within a few feet of the center of the channel, on either side. Under this plan several comparative observations must evidently be made to locate the point of maximum velocity. From a study of 24 different gagings, including both small and large streams, Prof. von Wagner found the value of the coefficient for this method to be given by the formula

$$C = \frac{V}{V_{ms}} = 0.705 + 0.01\, V_{ms},$$

V_{ms} representing the maximum surface velocity in the whole width of the stream (Min. Proc. Inst. Civ. Engrs., Vol.71, p.92).

For velocities of from 1 to 10 feet per second this formula gives a range of value for C from about 0.71 to about 0.81; when applied to the Vyrnwy gagings it was found to agree well with observed results, though usually giving somewhat too large a value, the error being in 20 cases out of 33 less than 3%, in

29 out of 33 less than 10%, and in no case as great as 16%.

It will be noticed that for a straight channel, of symmetrical section and approximately geometrical shape, in which case the center surface velocity is likely to be also the maximum surface velocity, or very nearly so, von Wagner's formula gives for ordinary velocities a coefficient lower than Prony's by say 0.06. On the whole it seems fair to suppose that for conditions (noticed in Art. 11) tending to depress the locus of maximum velocity in a vertical line, such for example as a relatively narrow or deep canal with vertical sides, Prony's formula will apply best, and in a natural river section von Wagner's.

Thus, in an artificial canal of say 800 square feet cross-section and an observed surface velocity at mid-stream of 4 feet per second, the discharge is likely to lie between
800 x 0.77 x 4 = say 2460 cubic feet per second,
and 800 x 0.87 x 4 = say 2780 cubic feet per second;
while in a natural river channel of 800 square feet cross-section and an observed maximum surface velocity of 4 feet per second, the discharge is likely to lie between
800 x 0.71 x 4 = say 2270 cubic feet per second,
and 800 x 0.81 x 4 = say 2590 cubic feet per second.

It has also been suggested that instead of surface velocity the central mid-depth velocity be observed, as being less variable than the former (in the Vyrnwy gagings the corresponding reduction coefficient ranged from about 0.83 to about 0.93, with a mean of 0.89); or, still better, that the mean velocity past the center vertical be observed (in the Vyrnwy gagings the coefficient for this case ranged from 0.87 to 0.99, with a mean of about 0.93). Such measurements would be less simple to make, however, than those on the surface, and with a stream in flood might be impracticable.

Still another method of quick approximate measurement has been proposed by Unwin, based on the fact that there must be in every cross-section two verticals (at least) past each of which the mean velocity is equal to the mean velocity for the entire section. A determination of that value in either vertical, or better in both, means for which will be described in a later article, would enable the discharge to be directly computed. It is hardly to be supposed that the location of either of these verticals could be predicted in general for irregular cross-sections, but for artificial sections they appear to come at about one-third the stream width each way from the center. Applied to 27 sets of Vyrnwy gagings this method gave the discharge for 8 sets within 1%, and for 24 sets within 5%.

15. Velocity Measurements in Each of a Series of Verticals. The great majority of stream gagings are made by observations in a series of vertical lines distributed across the stream, the immediate purpose being to obtain for each vertical the

mean of all the varying velocities past it from surface to bed. From these mean velocities the discharge can then be computed by methods to be explained later. The number of observations in each vertical is limited to as few as can be relied upon to give with satisfactory accuracy the mean velocity past the vertical, which is computed by assuming to hold a certain relation between the different velocities and applying, if necessary, appropriate reduction coefficients determined by independent observations. The law of distribution of velocities in a vertical assumes, therefore, prime importance, and an enormous amount of experimentation and study has been given to it.

16. Vertical Velocity Curves. If a stream be in technically steady flow, and in any vertical line the velocities past all points be conceived as simultaneously measured, the observation being long enough to average the pulsations, and the velocities be then laid off to scale from a vertical line, a curve will result which for brevity is called the "vertical velocity curve". The approximate geometrical shape of this curve is of some consequence, since the known relations holding for a given geometrical curve may suggest the number and desirable depths of observation points, and may lead to logical and simple rules for computing the mean velocity. The determination of the numerical values of constants in the equation of the curve is of minor importance, but if attempted can best be effected by the method of least squares.

Extensive observations for the express purpose of determining the representative shape of the vertical velocity curve have been made with double floats, but more frequently with meters, two methods being used with the latter:-

> 1st, and most commonly:- A single meter is held successively at different depths in the vertical, usually at each tenth of the total depth, and at each point long enough to average the effects of pulsations. This method is likely to require at least an hour for a single vertical, during which time an important change may occur in the mean velocity itself.
>
> 2nd:- A number of meters are used simultaneously,- if possible enough to occupy all the desired points in a vertical and thus give the entire curve at one observation. On the St. Clair river eleven meters were used by Haskell at one time, runs of ten minutes being made with the meters placed at each tenth of the depth.

A single set of observations in a vertical, especially if obtained from floats, or from short runs with a meter, may give a very irregular curve (Fig. 5), which, however, will become more and more regular as the number of sets is increased and averaged; and it is only from many sets combined that a typical curve results from which geometrical properties can properly be drawn (Fig. 6). The combining, for purposes of general

(see p. 20)

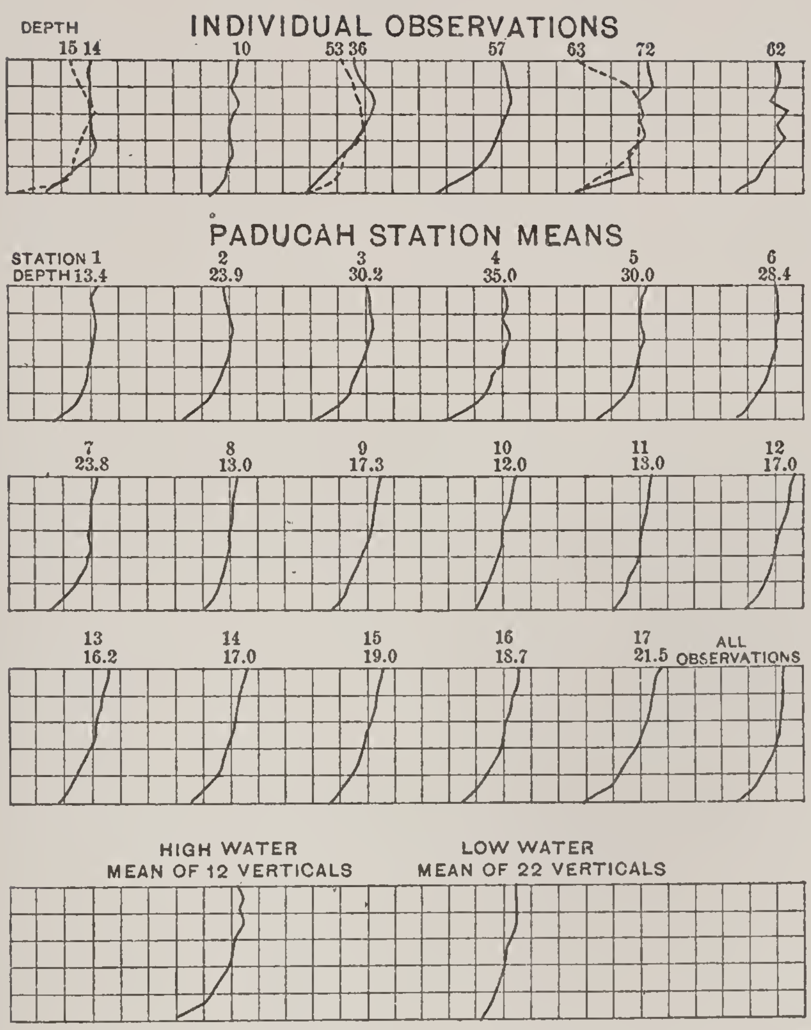


Fig. 5. Vertical Velocity Curves obtained by Miss. River Commission at Paducah.

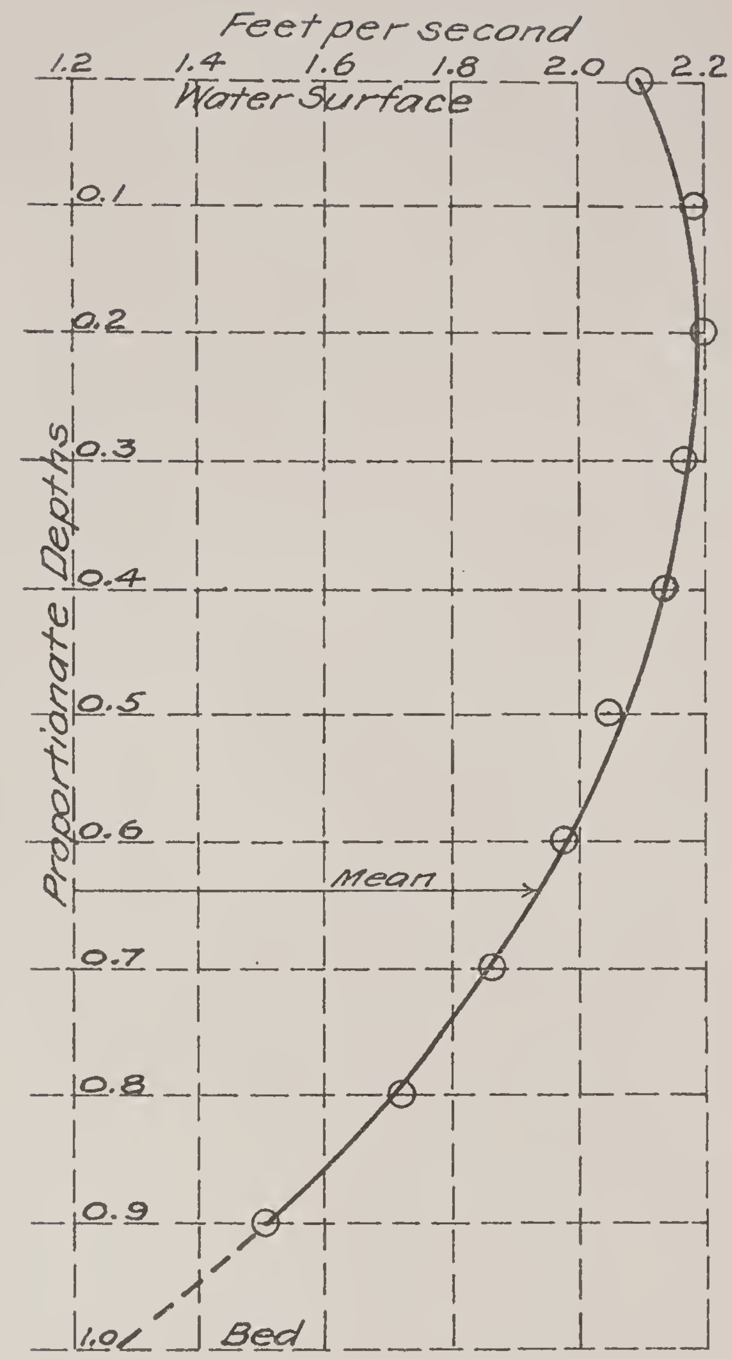


Fig. 6. Vertical Velocity Curve. Connecticut River at Thompsonville. Grand Mean from Ellis' Observations.

(Fig. 5. The stations in the cross-section are given by number, in order from the left bank, and the water depths at those stations. The upper row shows a few selected curves given by single sets of observations. Then follow curves obtained by combining and averaging for each station all the observations of a year, succeeded by a combination curve for all the stations taken together, with separate mean curves also for high water and low water conditions, respectively. See Starling on The Discharge of the Mississippi River; Trans. Amer. Soc. of Civ. Engrs., Vol. 34, p. 378.)

study of form, of vertical velocity curves obtained under varying conditions of depth and of mean velocity, or of position in the cross-section, is usually effected by determining for each individual curve its mean velocity and the relation of the velocity at each tenth of the depth to that mean, and then averaging the relative values thus obtained from all the different curves which are to be combined. Since, however, the shape of individual curves may vary more or less with the differences in the conditions just mentioned, it is open to question whether averaging may properly be attempted of curves for which those conditions are notably different.

There has been much diversity of opinion as to the geometrical curve best representing the distribution of velocities in the vertical, and investigators have variously found it to be the ordinary parabola with horizontal axis, a parabola with vertical axis and vertex at or below the bed, an ellipse with horizontal minor axis below the water surface, an hyperbola, a right line or a broken right line, a logarithmic curve, etc. Each of these may perhaps best fit some particular set of experiments, and in some cases two or more curves might about equally well fit the same set; but the most extensive series of observations, such as those of Humphreys and Abbot, Ellis, Cunningham and others, have led their authors to adopt the common parabola with horizontal axis as the typical curve, and this is now very generally assumed. Prof. von Wagner, from the investigation of 64 curves of small streams and large rivers, thought the law of distribution most accurately represented by a compound curve (Fig. 7; see Min. Proc. Inst. Civ. Engrs., Vol. 71, p. 89),- a parabolic arc extending from the axis, or line of maximum velocity (which he found to lie variously from 0 to nearly 0.3 of the total depth below the surface) down to about 0.8 of the total depth, but above and below this interval deviating inward from the parabola, with zero velocity at the bed.

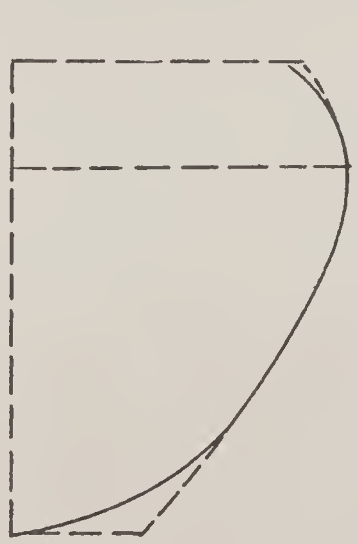


Fig. 7

17. Determination of Reduction Coefficients for Vertical Velocity Curves. The common necessity, on account of fluctuations of stage, of limiting the duration of any single gaging to a moderate time, say one or two hours, results in the general practice for large streams of taking observations at but one point in each vertical and, if necessary, applying a coefficient to reduce the observed velocity to the probable mean velocity past the vertical.

Although approximate values are known for the coefficient for all relative depths in the vertical likely to be used, it is desirable, in order to get the most accurate values, to make at each permanent gaging station a reasonable number of independent determinations of the coefficient by means of complete vertical curves observed at a number of different locations in the cross-section, and at different stages. The work can be done with one meter, but better and more rapidly with two. In the latter case one meter is held steadily at the relative depth proposed to be occupied in routine work, called the index point, and the other is held for five minutes, more or less, at each of the other points in the vertical (say at each tenth of the depth) necessary for a good determination of the vertical velocity curve. The practice of the Hungarian Hydrographic Department for deep streams has been to take observations for vertical velocity curves as follows: as nearly as possible at the water surface; then at depths of 0.25, 0.75, 1.50, 2.50, 4.00, and 6.00 meters; then at intervals of 2.00 meters and, finally, of 1.50, 1.00 and 0.50 meters; and, at the lowest point, as near as possible to the bed (Annales des Ponts et Chaussees, 1898-3 - p.301). From the complete curve, the ratio of the mean velocity in the entire vertical to the velocity at the index point is easily obtained.

18. Choice of Observation Points in Verticals. If the relations between the different velocities in a vertical were invariable, it is plain that we might measure the velocity at any convenient point in the vertical and from it determine the mean velocity from surface to bed. But the relation of the velocities at the different points to the mean for the vertical is not invariable, and varies more for some points than for others; moreover, the difficulty of accurately locating the observing instrument, and of making an accurate measurement with it when in place, is considerable at large depths in swift currents; and in time of flood, when there is much floating debris, a measurement much below the surface may be entirely impracticable. Consequently there is call for the exercise of judgment in choosing the point or points at which observations shall be made, and the following considerations are important:-

(a) In large and deep streams the vertical velocity curve is generally more nearly vertical from the surface to mid-depth than farther down, and failure to place the meter or floats exactly at the intended

depth is therefore likely to introduce less error into the result at or above mid-depth than below it. In gagings of the Niagara river at Buffalo in 1897-'98 all meter observations were made at 3/10 depth.

(b) From surface to bed the pulsations in velocity increase (Art. 10), and the error due to these is therefore less, for a short observation, at the higher levels.

(c) The ratio of the velocity at any particular depth in a vertical to the mean velocity for the vertical seems to be most constant in the vicinity of mid-depth, say in the middle third of the depth, and least constant near the surface and bed, especially in the upper and lower tenth of the depth. The error, therefore, in individual verticals, and in individual gagings when based upon a small number of verticals, due to deviation from the assumed ratio, is likely to be less if observations are made in the middle portion of the vertical than if above or below. Humphreys and Abbot in their extensive observations in the Mississippi found the constancy greatest at mid-depth. (Report upon the Physics and Hydraulics of the Mississippi River, p. 311). A study of mean velocity curves for seven streams considered in connection with the water supply of New York City showed the least variation between them at 6/10 depth (U. S. Geological Survey Water Supply and Irrigation Paper No. 76, p. 45).

(d) The small Price meter, at least, does not give accurate results, at velocities above 1.5 feet per second, if held nearer the water surface than one foot.

19. <u>Velocity Measurements at or near Surface.</u> The ratio of velocity at surface to mean velocity in a vertical shows less constancy than the ratio for almost any other position in the vertical, and surface measurements offer a corresponding disadvantage for accurate gagings. Still, the deviation is not excessive, and for stream reconnoissance or flood measurements the approximation obtained may be quite satisfactory. Furthermore, the greater ease and certainty in manipulation of the meter near the surface in swift currents, the opportunity for quickly raising it out of the water, if necessary for its protection from floating debris, and in case use of the meter is impracticable the easy substitution of surface floats, often make this method a convenient and even a necessary resort.

Its use for flood gagings was advocated by M. Charles Ritter in an article in Annales des Ponts et Chaussees (1886-2 - pp. 697, et seq). For measuring velocity he used a modified Darcy tube, and considered that current meter observations by Baumgarten, Harlacher and others, had shown that a reduction

coefficient of 0.85 applied to the velocity as measured say from 4 inches to 6 inches below the water surface would give the mean velocity in the vertical with an error not likely to exceed 5%; but in order to secure a safe or superior value for the discharge he proposed to use, not 0.85, but 0.90, and believed that the result would generally be in error less than 10%.

The advantage of surface measurements in high stages of streams, when the rapid fluctuations preclude the slower but more accurate method of observation at several points in each vertical, was also pointed out by Harlacher and Richter (Min. Proc. Inst. Civ. Engrs., Vol. 91, pp. 397 et seq). For 28 series of verticals obtained in gagings in the Danube and in Bohemian rivers, comprising about 300 individual curves, they found the average value of the reduction coefficient 0.84, with a range for the different series (not for individual curves) from 0.79 to 0.91. They quoted also similar observations by Swiss and Dutch engineers showing mean values for the coefficient of 0.835 and 0.87, respectively.

Hoyt and Grover give figures for between 800 and 900 vertical velocity curves measured by engineers of the U. S. Geological Survey, at about 70 different gaging stations, showing the mean value of the reduction coefficient to range for different stations from 0.78 to 0.95, with an average for all stations of 0.85; and an elaborate measurement of the Susquehanna at Harrisburg, November 2, 1903, by engineers of the same survey, showed a mean coefficient, to be applied to velocities observed <u>one foot below the surface</u> of 0.85 for 14 verticals, with a range in individual verticals from 0.76 to 0.91 (Eng. News, Jan. 14, 1904.)

Apparently those conditions which tend to depress the locus of maximum velocity in a vertical (Art. 11) will tend also to increase the numerical value of the coefficient, which should therefore be higher for relatively narrow, deep channels, than for wide, shallow ones; and Grunsky has given a graded set of mean values for the coefficient, diminishing from 1.03 for streams whose width is 5 times the average depth, to 0.91 for streams whose width is 30 times the average depth, and to 0.82 for streams whose width is 100 times the average depth. (Engineering Record, Mar. 7, 1896). The experimentally determined coefficient would vary somewhat accordingly as the so-called surface velocity was measured close to the surface, as with a thin block of wood, or several inches, or perhaps a foot below the surface, as with some current meters, and published data do not always make clear the exact depth of the observation; but as a fair general approximation for river cross-sections we may write:-

$$\frac{\text{Mean velocity in any vertical}}{\text{Velocity at or within 1 foot of surface in same vertical}} = 0.85$$

20. <u>Velocity Measurements at Probable Depth of Mean Velocity past Vertical</u>. Assuming the vertical velocity curve to be an ordinary parabola with horizontal axis, the locus of mean velocity will theoretically vary from 0.58 depth, with maximum velocity at surface, to 0.65 depth, with maximum velocity at 3/10 depth. It appears, therefore, that the thread of mean velocity in any vertical is likely to be found, in general, in the near vicinity of 6/10 depth, and this accords with mean results actually obtained from very many different series of experiments under widely varying conditions.

It is important to notice, however, that while 6/10 depth is a good <u>average</u> value for the locus of the mean velocity in verticals, yet that for particular cross sections, and still more for particular vertical curves of any single cross-section, there may be wide departures from that value. Taking the average relative depth of mean velocity for all verticals at a single gaging station, and comparing these averages for different streams and stations, the range will be well covered by 0.55 - 0.65; and yet, for the Mississippi at Carrollton, by the observations of 1883, the relative depth was 0.66 at low water and 0.77 at high water, while it varied from 0.3 to 0.8 in individual curves obtained in 1882 (Trans. A.S.C.E., Vol. 34, p. 384).

For the Niagara, by the measurements of 1891-92, the relative depth varied in different verticals from 0.47 to 0.68 in one cross-section, and from 0.50 to 0.64 in another, although the general average was 0.58 for one cross-section and 0.60 for the other (Eng. News, March 2, 1893).

At the Cornell flume, in 1900-01, Murphy found the relative depth in different individual experiments to vary from 0.50 for a water depth of 6 or 8 inches, to 0.73 for a water depth of 8 or 9 feet, although the average of 31 experiments was 0.64 (U.S.G.S. Water Supply and Irrigation Paper, No. 95, p. 99).

In an elaborate measurement of the Susquehanna at Harrisburg by engineers of the U. S. Geological Survey, November 2, 1903, the relative depth of the thread of mean velocity varied among the twenty verticals from 0.51 to 0.72, although the average of all was 0.61 (Eng. News, Jan. 14, 1904); and Hoyt and Grover give data for over seventy gaging stations of the U. S. Geological Survey, covering about 900 vertical velocity curves, the mean position, for individual stations, of the thread of mean velocity varying from 0.58 depth to 0.73 depth, with an average for all stations of 0.61, the error in the mean velocity past the vertical, when assumed to be given by the observed velocity at 6/10 depth, not exceeding 6% for the mean of any station, and averaging zero for all the stations taken together.

Von Wagner has given data for nine sets of curves (64 in all) for the Danube, Rhine, Elbe, Weser and Oker rivers, ranging from 425 meters down to 14 meters in width, showing an average relative depth of the thread of mean velocity for all of 0.597, the range between different sets being only from 0.58 to 0.62, although the locus of maximum velocity varied from the surface down to 0.25 depth (Min. Proc. Inst. Civ. Engrs., Vol 71, p. 90).

It is evident then that, although a point measurement at 6/10 depth is likely to give a close approximation to the mean velocity past the vertical for an ordinary natural channel, yet, if the best results are sought, some discrimination should be used as to varying the relative depth of measurement for different channels, and even for different parts of the same channel; and it is best that, by special experiments at each gaging station, the proper relative depth should be found for different verticals and river stages, or, what amounts to the same thing, that the proper reduction coefficients to apply to observations at 6/10 depth should be determined. The practice of measuring at 6/10 depth in large rivers is common with government engineers in this country, a reduction coefficient sometimes being employed and sometimes not, according to circumstances.

It is well to remember that those conditions (noticed in Art. 11) which tend to depress the locus of the thread of maximum velocity in the vertical tend also to depress that of mean velocity, and vice versa; consequently, in deep, narrow channels we may expect the thread of mean velocity to be at a relatively lower level than in shallow, wide ones. Unusual roughness of the bed, making the decrease of velocity towards the bed more pronounced than otherwise, also tends to lower the thread of mean velocity. As the result of elaborate studies for the U. S. Geological Survey (Water Supply and Irrigation Paper, No. 95, p. 137). Murphy concluded that for an approximately straight, regular channel, with few obstructions,

(a) "In a broad, shallow stream from 3 to 12 inches in depth, and having a sand or fine gravel bed, the thread of mean velocity is from 0.50 to 0.55 depth below the surface."

(b) "In broad streams from 1 to 3 feet in depth, and having gravelly beds, the thread of mean velocity is from 0.55 to 0.60 of the depth below the surface."

(c) "In ordinary streams where the depth varies from about 1 to 6 feet, the thread of mean velocity is about 0.6 below the surface."

For the Merrimack measuring flume, Lowell, width 48 feet, water depth about 10 feet, Lynch and Wheeler (M.I.T. 1894, 1895) found the mean relative depth to be 0.67 for the thread of mean velocity.

With the maximum velocity below 1/3 depth, as occurs in ice covered or other closed channels flowing full, and near the side of some open channels, there may be expected <u>two</u> loci in a vertical for the thread of mean velocity, and <u>in open channels</u> for such curves a fair approximation to the mean velocity may be obtained by an observation at either 5/8 depth or 1/10 depth.

21. <u>Velocity Measurements at Mid-Depth in Vertical.</u> Since the mean velocity in the vertical frequently occurs at a relative depth differing considerably from 6/10, and since for close results a reduction coefficient must then be applied to velocities observed at 6/10 depth, and since there has been claimed to be greater constancy in the velocity at mid-depth than at 6/10 depth, engineers have frequently preferred to make their measurements at the higher level.

Humphreys and Abbot considered their extensive observations in the Mississippi to prove the ratio of mid-depth velocity in a vertical to mean velocity in the same vertical to be a "sensibly constant quantity for all practical purposes" with a mean value of about 0.98, but their conclusions have not been confirmed by later observers. Starling states (Trans. A.S.C.E., Vol. 34, p. 382), that the measurements of the Mississippi River Commission in 1882 showed "the ratio between the mean of the mean velocities in all the verticals at each observation point and the mean of the mid-depth velocities in the same verticals" to vary from 0.94 to 0.98, the mean of all being about 0.96.

To test the constancy of the mid-depth velocity Cunningham made special experiments in the Ganges canal, measuring the same mid-depth velocity forty-eight times in quick succession with double floats and twelve times with current meter, obtaining a ratio of mean to mid-depth velocity ranging from about 0.92 to about 1.08 (Min. Proc. Inst. Civ. Engrs., Vol. 71, pp. 15, 19).

Observations below the Vyrnwy reservoir, by Ellis in the Connecticut river near Thompsonville, by the Mississippi River Commission, as quoted by Starling, by the U. S. Geological Survey in the Esopus and six other streams considered in connection with the water supply of New York City, by Wheeler and Lynch in the Merrimack canal at Lowell, and others, show mean values of this ratio for the various gaging stations ranging from 0.92 to 0.98.

Observations by Allen and Griffin (M.I.T. 1908) as to the comparative constancy of velocities at successive tenths of the depth in selected verticals in four cross sections (Charles River at two points, Sudbury Aqueduct, and Hamilton Flume, Lowell) show for three of these greater constancy at mid-depth than at any other depth. It seems to the writer that such published information as is available indicates some advantage in constancy for mid-depth, but that the advantage is not great over other depths in the middle third of the vertical, and that the constancy is not such as to preclude the necessity in close

work of using specially determined reduction coefficients. For approximate results, however, we may assume:-

$$\frac{\text{Mean velocity past a vertical line}}{\text{Mid-depth velocity past same vertical}} = 0.95$$

22. Single-Point Velocity Measurements at Sundry Depths in Verticals. Whether 6/10 depth, mid-depth, or the surface be chosen for single-point measurements, it has been shown that a reduction coefficient must usually be applied for accurate results; consequently, other depths than those may answer about equally well, and have occasionally been employed. It has already been mentioned (Art. 18) that all meter observations in the 1897-98 gagings of the Niagara river were made at 3/10 depth (Jour. Wes. Soc. Engrs. 1899, p. 459); and Starling states that in some of the Mississippi river gagings observations have been taken at various depths, as might be most convenient, appropriate reduction factors being used (Trans. A.S.C.E., Vol. 34, p. 383).

23. Velocity Measurements at Two Points in Verticals. Assuming the typical vertical velocity curve to be a common parabola with horizontal axis, several formulae may be derived for expressing the mean velocity past the vertical in terms of two measured velocities. Cunningham presented four such formulae in his paper on experiments in the Ganges canal, two of which are as follows (Min. Proc. Inst. Civ. Engrs., Vol. 71, p. 18):-

(a) $$\text{Mean V past vertical} = \frac{\text{V at surface} + 3\ (\text{V at } \tfrac{2}{3} \text{ depth})}{4}$$

(b) $$\text{Mean V past vertical} = \frac{\text{V at 0.211 depth} + \text{V at 0.789 depth}}{2}$$

$$\text{or approximately} \quad = \frac{\text{V at } \tfrac{2}{10} \text{ depth} + \text{V at } \tfrac{8}{10} \text{ depth}}{2}$$

The first has the advantage that the velocities are measured at the highest levels possible for such a formula. With the second the computation involved is simple; with floats it would be possible to run a connected pair whose common velocity should be the mean in the vertical; and with the current meter the specified locations for the instrument are definite. Prof. Von Wagner applied this formula to a number of measured curves for the Weser, Elbe, Rhine, Danube, etc., and found it in most cases to agree perfectly, the differences never being more than 2.5% (p. 89 of above reference). Hoyt and Grover give the results of applying the formula to a total of 461 vertical velocity curves determined since 1905 at 33 different gaging stations of the U. S. Geological Survey, from which it appears that the mean correction needed for any station to make the values for mean velocity past the vertical by formula agree

with the values given by the curves themselves in no case exceeded 3%, and for the whole number of stations taken together averaged practically zero. (River Discharge, p. 50).

Assuming, as a rough approximation, the vertical velocity curve a straight line from surface to bed, the mean of the nominal top and bottom velocities is sometimes taken as the mean past the entire vertical. The approximation is seldom warranted, however; observations cannot be made strictly at the surface or bed with meters, and the distance from surface and from bed will vary with the construction of the meter; the immediate vicinity of the bottom is the poorest of all positions in the vertical for velocity measurements, because of irregularities of the bed, eddies and pulsations; and correction coefficients for this method are neither well known nor likely to be constant. The mean of top and bottom measurements should therefore seldom be employed, unless possibly for wide, shallow streams having a smooth bed.

24. Velocity Measurements at Three Points in Vertical. If velocities be measured at three points in the vertical, say at or near the top, middle, and bottom, their values plotted to scale, and a smooth curve drawn through the points thus located, a rough approximation to the vertical velocity curve will be obtained, and with no particular assumption as to its geometrical shape the mean velocity past the vertical can be found by determining the area of the curve by planimeter or otherwise and dividing the area by the total depth.

If the curve be regarded as approximately represented by a broken line, then, by the trapezoidal rule, letting T, M, B, and V represent the three measured velocities and the mean velocity, respectively,

$$V = \frac{T + 2\,M + B}{4}$$

If, however, the curve be assumed to be better represented by a parabolic arc passing through the observation points and having a horizontal axis, then

$$V = \frac{T + 4\,M + B}{6}$$

The same objections hold here to the use of top and bottom velocities that were mentioned in Art. 22, and a slight error is bound to result from the fact that velocities cannot be measured strictly at the surface or bed; but if for the above observation points there be substituted 2/10 depth, 8/10 depth, and the probable depth of the thread of mean velocity (ordinarily 6/10 depth), then, with the above assumption of a parabolic arc, we shall have, very closely,

$$\text{Mean V past vertical} = \frac{\text{V at } \frac{2}{10} \text{ depth} + 2\,(\text{V at say } \frac{6}{10} \text{ depth}) + \text{V at } \frac{8}{10} \text{ depth}}{4}$$

25. Determination of Mean Velocity past Vertical from Curve. The most accurate method of determining the mean velocity past the vertical with current meter, especially where artificial or unusual natural conditions prevent the ordinary distribution of velocities, is to observe velocities at enough points in the vertical so that when they are plotted to scale the vertical velocity curve may be drawn through the points thus determined. The area of this curve may be found by planimeter or by any other suitable method, and the area divided by the depth gives the mean velocity. The scale for velocities should be made no larger than appears necessary, and an irregular curve may be drawn so as to pass through all the points, or a smoother curve may be interpolated, but the former method seems to the writer the better. Since observations cannot be taken directly at the surface or bed, the curve will have to be extended by judgment beyond the plotted points, but its probable direction is usually evident. It may be well to remember, though, that the velocity probably diminishes pretty rapidly in the immediate vicinity of the bed, perhaps more rapidly than the general trend of the curve would indicate.

It is well to occupy at least five points in the vertical in any case, and there is a possible advantage, for the purpose of comparing different methods of computation, in including 2/10, 5/10, 6/10 and 8/10 depth; frequently, observations are made at each tenth of the total depth, and sometimes on deep streams, at still shorter intervals (see Art. 17).

The time required for applying this method throughout a stream gaging usually limits its application to channels of moderate size, or to certain selected verticals in a large stream, the purpose in the latter case being to determine, after sufficient repetitions, proper average values for reduction coefficients.

26. Integration of Velocities Past Vertical. If the current meter be moved at a uniform rate through a vertical, the total number of revolutions of the wheel and the total time interval being noted, the average number of revolutions per second may be found and, from the rating of the meter, the corresponding velocity of the water. This will be approximately the mean velocity past the vertical. There are certain minor defects in this method, however, as follows:-

(a) The construction of meters prevents their quite reaching the bottom of the vertical, with some types within six inches or a foot, or even more, and the slow moving velocities near the bed are not included in the result, which is therefore too large. In the gaging of the Susquehanna at Harrisburg, November 2, 1903, by engineers of the U. S. Geological Survey, the integration method gave about 2% higher discharge than that by vertical velocity curves.

(b) The relation between revolutions of the meter and velocity of the water is not represented by a right line, but by a slightly curved line; consequently, the greater the variation of the velocities past the vertical, the less accurately the mean speed of the meter wheel corresponds to the mean velocity of the water.

(c) The vertical movement of the meter itself affects the indicated velocity, decreasing it in the case of the Fteley meter, but increasing it in the case of the Price or any other type of meter adjusted so that it is free to head in the direction of the resultant velocity. The relative error is greatest in slow currents, and to make it negligible the vertical speed of the meter must be made small.

(d) In slow currents the meter may, in its vertical motion, especially near the bed, encounter velocities so low that its indications are unreliable or that it entirely ceases to revolve.

(e) It is difficult to obtain, without a complicated outfit, or at least the aid of an assistant, such regularity of vertical motion as to make negligible the error due to variable speed (see description of Harlacher's apparatus for moving a meter with uniform velocity: U.S.G.S. Water Supply and Irrigation Paper, No. 95, pp. 38, 39).

Nevertheless, in spite of these defects, the method of vertical integration, in the hands of a judicious and practiced observer, may give a high degree of accuracy, even without any special contrivance for ensuring uniformity of motion. Where conditions are poor, as where the channel is crooked, or irregular in outline, or the surface is frozen or otherwise obstructed, thus preventing the usual relations between velocities past the vertical; and where time is wanting for complete point measurements in verticals,- under such circumstances, if the velocities have not too low a range, the simplicity and comparative speed of vertical integration may make it of great service. It is also useful as a check upon results obtained by other methods. The single operation of lowering the meter to the bed may give at once the depth and the approximate mean velocity past the vertical. The integration may be made either downward or upward, and very commonly the two trips are combined in one continuous observation.

27. Method of Measurement used by Hydrographic Department of Hungary. The Hydrographic Department of Hungary has employed an ingenious method of determining the mean velocity past verticals, called the "detailed" method (Annales des Ponts et Chaussees, 1898-3, No. 40). It combines with the speed of vertical integration substantially the accuracy of point measurements, while giving even more complete information than the

latter as to the variation of velocities past the vertical and as to the shape of the curve. The meter is lowered and raised at a slow but regular speed by a cable and windlass. As the windlass turns it unrolls at a reduced, but proportionate rate, a band of paper from a specially arranged chronograph, the length unrolled up to any given moment measuring the vertical distance through which the meter has sunk or risen in the water. The chronograph impresses on the paper band characteristic marks at the expiration of each half second of time and of each revolution of the meter wheel.

The procedure resembles that in integration, the meter being lowered to the bed and then raised to the surface, thus giving two complete determinations for the vertical. While regularity of motion is sought, it is far less essential than in ordinary integration. Intervals of two centimeters height on the paper, ordinarily corresponding to twenty centimeters of actual vertical distance in the water, are marked off, and opposite each such interval the chronograph record enables one to compute the meter speed in revolutions per second, from which in turn the corresponding velocity of the water may be found from the rating of the meter. The data are then known for plotting the vertical velocity curve, which can be traced with greater or less refinement by varying the lengths of the intervals into which the record is divided. The extremes of the curve at surface and bed must be drawn by prolongation as in ordinary point measurements. While traversing a vertical in 12 or 14 meters depth of water would require nearly or quite an hour by point measurement, it is accomplished in about five minutes by the "detailed" method.

28. Measurement of Mean Velocity past Vertical by Rod or Tube Floats. By the use of a wooden rod or closed metal tube (usually of small diameter) so weighted at the bottom as to float with axis vertical and but little exposure above the water surface, and reaching as near the bed of the channel as circumstances will permit, a close approximation may be made to the mean velocity from surface to bed along the path traversed by the float, if the depth be uniform, as in an artificial channel. The method is frequently applied also to natural channels, but with less satisfactory results.

Inasmuch as the float cannot be allowed to reach quite to the channel bed, it is not acted upon by some of the slow moving water near the bed, and therefore tends to move faster than it would do if more deeply submerged. But it does not strictly move with the mean velocity of the water, even for the depth of actual immersion, for the following reason: The float is subject to pressures proportional to the square of the relative velocities of the water at different points with respect to it, beyond a certain speed these pressures presumably being exerted simultaneously against the up-stream side of the float in the upper part of its length and against the down-stream side in the lower part of its length; and the float will not reach its

full normal speed until equilibrium has been reached between these different pressures. It is evident that this equilibrium may exist, however, when the speed of the float is somewhat different from the mean velocity of the water for the depth of immersion, the latter velocity being simply the arithmetical mean of all the different velocities from the water surface to the bottom of the tube.

As the result of mathematical investigation, making the ordinary assumption of the vertical velocity curve being a parabola, Cunningham concluded that the float velocity would always be somewhat less than the mean velocity of the water for the depth of immersion, and that to measure the mean velocity past a vertical the float should be immersed only about 0.94 of the water depth (Min. Proc. Inst. Civ. Engrs., Vol. 71, p. 22).

Elaborate experiments by Francis at Lowell in the Tremont measuring flume (widths in different series of experiments about 27 and 13 feet, respectively, and water depth 9 feet, more or less) showed, however, that the mean velocity of a series of floats for a complete gaging nearly always exceeded the mean velocity of the water past the entire cross-section as found by weir measurement, and led to the following correction coefficient:-

If V = true mean velocity past vertical from surface to bed,

V_o = observed velocity of tube,

d = mean water depth along path of tube.

d' = depth of immersion of tube,

then $$C = \frac{V}{V_o} = 1 - 0.116\left(\sqrt{\frac{d-d'}{d}} - 0.1\right)$$

It will be noticed that the value of this coefficient rises to unity only when the depth of float immersion reaches about 0.99 of the water depth. Probably this formula would not strictly hold for other than rectangular channels, nor in any case where there was an abnormal distribution of velocities past the vertical; nevertheless it has been widely applied to various conditions in practice.

In Francis' experiments the ratio $\frac{d-d'}{d}$ did not exceed 0.12, and to apply his correction formula to cases in which the relative clearance below the float much exceeds 0.12 of the depth involves an error that is not well known. On repeatedly running at short intervals a series of floats of different lengths, from 7 to 11 feet, over the same course, in water about 12 feet deep, Brown found that, assuming Francis' coefficient correct for 5% clearance below the float,

$\left(\frac{d-d'}{d} = 0.05 \right)$, the numerical value of the coefficient then being 0.986, the successive values best representing the results of his own experiments were as follows (Wis. Engr., Vol.6):-

for 10% clearance C = 0.969
" 20% " C = 0.942
" 30% " C = 0.919
" 35% " C = 0.908

Using tube floats of different lengths, which were run alternately with standard long tubes submerged about 95% of the water depth, Dort and Faulkner, M.I.T. '09, made a large number of observations similar to those by Brown. Their runs were made in two canals at Lowell,- the Merrimack, about 48 feet wide, with 10 or 11 feet depth of water, observed velocities ranging between 2 and 3 feet per second; and the Boott, about 41 feet wide, with 8 or 9 feet depth of water, velocities ranging mainly between 4 and 5 feet per second. For clearances less than 35% their mean coefficients for the Boott canal agreed best with those by Francis' formula, in no case differing from the latter by more than 1%; but for the Merrimack they agreed much better with Brown's values than with Francis', being practically identical with the former throughout.

29. Proper Spacing of Verticals. The choice of horizontal intervals at which observations shall be made is more or less arbitrary, but should have general reference to the accuracy to be aimed at in the result, the steadiness of the stream, and the time likely to be required for a complete gaging. The judgment may be aided by having in mind the transverse velocity curve (Art. 32), since such spacing as would determine this satisfactorily should be otherwise acceptable. This principle would lead to observations being taken at conspicuous points of the transverse profile of the stream bed, relatively closely near the banks, and farther apart where the velocity changes slowly and regularly. Computation of discharge without plotting is facilitated by spacing velocity stations as well as soundings at regular intervals, and the general instructions of the U. S. Geological Survey (Water Supply and Irrigation Paper No. 94, page 19) call for measurement of velocities in each vertical in which a sounding is taken, except where the change in velocity is small, and there in alternate verticals. This would lead to intervals ranging from 1 foot to say 50 feet, according to the size of stream (Art. 5).

The practice of the Hydrographic Department of Hungary is not to take intervals greater than from 20 to 40 meters, even on a stream as large as the Danube. On the Connecticut, where from 1,000 to 1,500 feet wide, Ellis used intervals of about 100 feet. On the Niagara, width about 1,800 feet, Haskell

had meter stations about 80 feet apart. About 1890 the practice on the Mississippi river was to space meter stations 300 feet apart, but the interval was afterward reduced to 150 feet; comparative computations of discharge were made by Captain Townsend in 1893, using the observations at 300-foot and 150-foot intervals, respectively, with a resulting difference ordinarily less than 1% and never exceeding about 3%.

30. "Flanking" Method of Measuring Velocities. As a check upon measurements otherwise obtained, a method known as "flanking" has been used in the Mississippi river, resembling in a rough way, in principle, the "detailed" method described in Art. 27, but involving a horizontal instead of a vertical motion of the meter. The meter is submerged 15 or 20 feet, so as to be below disturbance from the launch, which is then worked slowly across the river, being kept as closely as possible on the established transverse range, head up-stream, until the opposite bank is reached, when a return trip is made, the two trips constituting one complete observation. At regular distances, fixed in advance by means of shore ranges, readings are taken of elapsed time and revolutions of the meter, from which may be determined the average resultant velocity of the water in each section of the path of the meter. The observed velocity may be corrected for the lateral motion of the boat as follows:

If V' = velocity given by meter,

V'' = velocity due to lateral motion of boat,

V = absolute velocity of water,

then $V = \sqrt{V'^2 - V''^2}$

The mean of the results obtained in opposite trips is adopted.

31. Continuous Integration Over Entire Cross-Section. Where one may operate the current meter from a solid support, such as a beam spanning the channel, or by wading, it is often practicable to integrate velocities over substantially all parts of the cross-section in one operation, and thus to determine at once its mean velocity, and excellent results are often obtained in this way in a very brief time. The wetted cross-section should be conceived as subdivided by imaginary lines into vertical or horizontal strips of equal width, and the motion of the meter through these so directed that, as nearly as possible, equal elementary areas shall be centrally traversed in equal times. If the strips are vertical, the motion is often a diagonal one, the meter being started at the surface at one side of the channel and moved obliquely down at a slow, uniform rate to the bed, returning thence obliquely to the surface, thus covering one strip, and so on through the remaining strips to the farther side of the channel.

On the lines of the aqueducts supplying the Boston metropolitan district with water are permanent gaging stations at which frequent current meter measurements are made by a continuous integration of consecutive 6-inch horizontal strips. The path of the meter through these successive strips is mechanically controlled (see Eng. News June 12, '02, p. 488, Fig. 3), and the speed of motion is determined by the operator, who is aided by a metronome beating seconds and is checked at intervals by an assistant with a stop watch. The downward and return upward integrations usually vary somewhat in the results, but the average error of their mean is believed not to exceed 0.25%. If the water level is such as to give a surface strip of materially different width from the standard, each lower strip is integrated separately, the respective mean velocities of the strips are plotted to scale, and the extension of the resulting curve gives the probable mean velocity of the surface strip.

32. <u>Transverse Velocity Curve.</u> If the mean velocity past each of a series of vertical lines, regularly distributed across the section, be plotted to scale, in its proper position as an ordinate from a horizontal line representing the water surface, there will be determined a curve which for convenience will be called a "transverse velocity curve", and which is often employed in the computation of discharge. In general the ordinates to this curve will be greatest in the deepest part of the channel, diminishing as shoal water is approached, or the banks are neared, and very rapidly close to the banks.

33. <u>Conception of Discharge as Represented by a Solid.</u>

If a differential area in the cross section be imagined as the base, and the velocity past that area the altitude, the solid constructed with these two dimensions will represent a differential portion of the discharge. If the entire cross-section be similarly treated, the resulting solid will represent the entire discharge. The method which determines the volume of this solid with the closest approximation, determines the discharge most accurately. The computation of the discharge can often be made without the aid of any plot, as where soundings and their velocities are measured at regular intervals across the stream; but even then it is an advantage, as a check against gross errors, and in other cases it is a necessity, to plot the cross-section, if otherwise than rectangular, the transverse velocity curve, and sometimes a transverse discharge curve.

34. <u>Subdivision of Solid into Slices.</u> The solid representing the discharge is conceived to be divided into slices, usually of equal width, by parallel planes cutting it in one of three directions;-

(a) Parallel to plane of water surface; i.e. horizontal.
(b) Parallel to plane of cross-section; i.e. vertical, and transverse to stream.

(c) Perpendicular both to plane of cross-section and to that of water surface; i.e. vertical, and longitudinal with stream.

Usually the slices are taken of definite width, varying say from one foot upward according to circumstances, and some suitable formula is applied to finding their approximate volume. The curved surfaces are not made up of straight-line elements, and the prismoidal formula is not strictly applicable. Frequently the volume of a slice is taken as the area of a mid-section multiplied by the thickness of the slice. The thinner the slices the more accurately may the computation be made, and in some methods they are even treated as of only differential thickness. No more refinement is really necessary, however, than shall be consistent with the completeness and probable accuracy of observations.

Case (a) Each slice, except the bottom one, in natural channels, will have three plane faces, with sides and down-stream end curved. The top and bottom face will each be limited by a horizontal velocity curve; but velocities are not usually measured at such depths as directly to define those curves, and this method is not therefore commonly used.

Case (b) The slices will in general have three plane faces, with sides and bottom curved. The two principal faces of each slice, formed by the parallel cutting planes, will be bounded by contours of equal velocity, which can be traced on the cross-section in the case of numerous and well distributed point measurements only. Their areas can readily be measured by planimeter. The residual volume at the apex of the solid is likely to be of less height than the slices, and its contents can be only roughly computed. This method is described by Unwin (Hydraulics, p. 289), but is probably not often used.

Case (c) Except next the sides of the channel each slice will have plane faces for its base (considered as a part of the cross-section), top and two sides, with a curved surface for the remainder of its exterior, this curved surface not being of any definite geometrical shape. It is this arrangement of slices upon which practical computations of discharge are most often based, although various procedures are adopted for the details of the work.

35. Harlacher's Graphical Method of Computing Discharge. Referring to Fig. 8 it will be seen that, if D is the depth in a particular vertical, and V is the mean velocity past that

vertical, then the discharge past a strip of the cross-section dx in width will be V D dx. If, therefore, one were to lay off a series of ordinates from the line representing the water surface, making each of a length equal to its respective V D, and extending the operation from shore to shore, the area of the curve thus formed would evidently represent the total discharge.

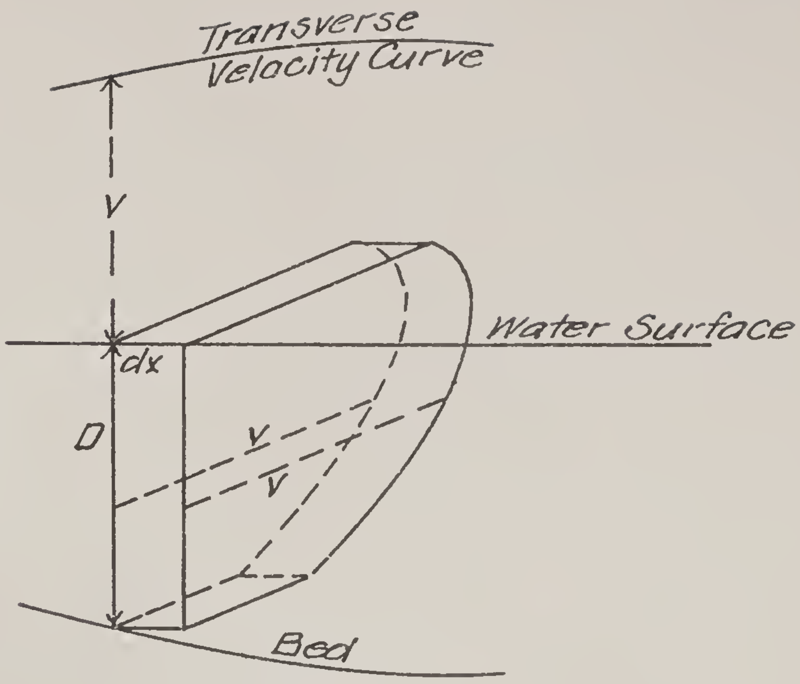


Fig. 8.

Harlacher has shown a graphical method of constructing this curve ("Die Messungen in der Elbe und Donau, und die hydrometrischen Apparate und Methoden des Verfassers", Leipsig, 1881; see also "Hydraulique", par A. Flamant, Paris, 1891, p. 359) the determination of the discharge being thereby reduced to the planimetric measurement of an area (Fig. 9):- If, as before, V represents the mean velocity past any vertical D, then, as before, V D dx represents a differential portion of the discharge, and the entire discharge Q = $\int$V D dx, the integration extending from one bank to the other. Lay off a convenient arbitrary distance K, make E C = V, and draw a line from C parallel to A B, cutting E B produced at P. By construction V D = (E P) K; consequently $\int$V D dx = K$\int$(E P) dx = K multiplied by the area of the discharge curve which is the locus of all points P determined as above.

If the area of the discharge curve be measured by planimeter in square inches, and the length of K in inches, velocities being plotted to a scale of S feet to one inch, horizontal distances to a scale of S' feet to one inch, and depths to a scale of

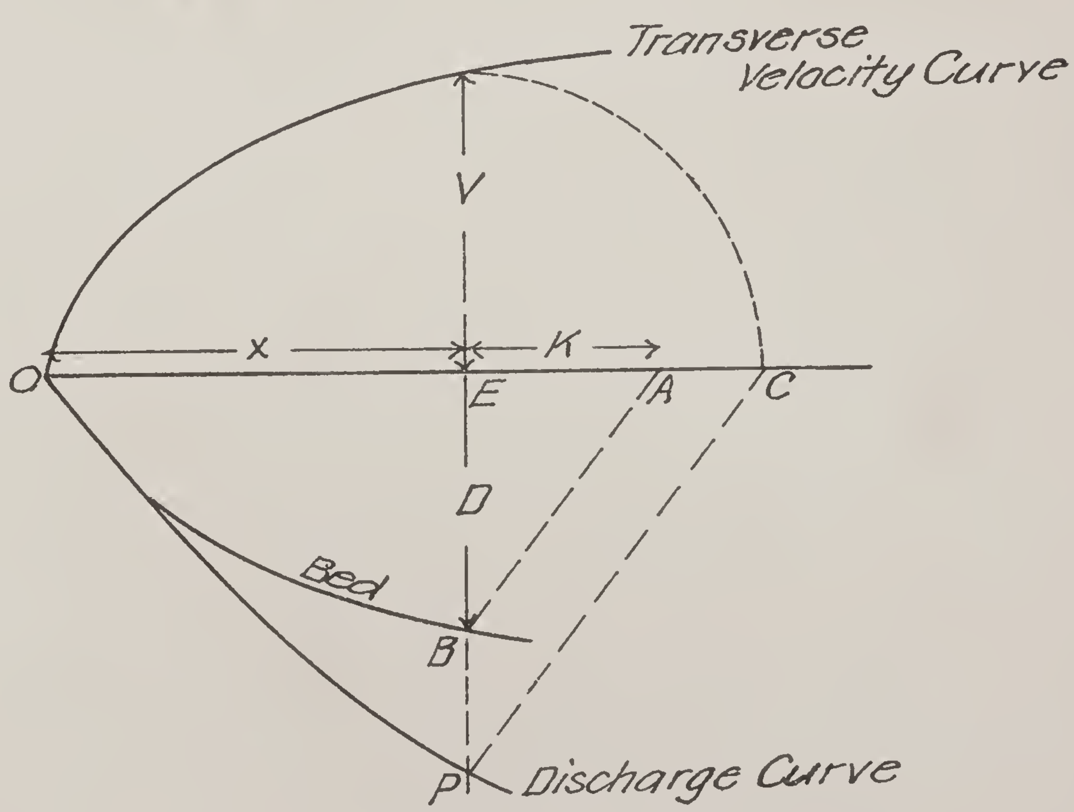


Fig. 9.

S" feet to one inch, then the measured area must be multiplied by K S S' S" to give the discharge in cubic feet per second.

36. Procedure in Computing Discharge for Irregularly Distributed Verticals, as in Rod or Tube Float Measurements. The current meter can be very exactly placed in selected verticals, ordinarily spaced at equal intervals across the channel, and the drawing of a transverse velocity curve is not always then necessary for computing the discharge, although it is always useful as a check against error. Floats, however, seldom pass the upper and lower cross ranges, which define their run, at the same distance out from the reference line on the shore, and the mean positions of successive floats are likely to be separated by more or less irregular intervals; moreover, closely adjacent runs often give materially different velocities. Therefore, and especially for narrow channels, in which the floats are apt to be run at rather short lateral intervals, the drawing of a transverse velocity curve is often a practical necessity.

The mean position of a float must usually be taken as fixed by the mean of its distances out at the crossing of the upper and lower ranges. Whatever its path between those ranges, whether normal to them, oblique, or curved, its average velocity normal to the ranges,- and it is only that component which is desired for computing discharge - is given by dividing the normal distance between the ranges by the elapsed time in making the passage. This velocity may then be reduced to mean velocity in the vertical by applying the proper correction coefficient.

If, now, the values obtained for mean velocity in the various verticals be plotted to scale, in their relative positions, as ordinates from the straight line representing the transverse profile of the water surface at its mean position during the gaging, points will thus be fixed for drawing the transverse velocity curve. A reasonable curve can seldom be drawn, however, which shall pass through all the plotted points, since closely succeeding ordinates may vary materially in length on account of irregularities of current, in which case a mean curve must be interpolated. This may first be drawn roughly by eye, following the general trend of the points, and then adjusted and redrawn through the successive groups into which the points either naturally fall or may arbitrarily be assigned by arranging that in each group the plus and minus ordinates with respect to the curve shall balance (Fig. 10). The plus values in a group may be set off successively on the edge of a strip of paper, and similarly the minus values, and the balancing is thus easily tested.

The smaller the number of points taken in a group, the less smooth the curve will be, but if used with discretion, as an aid to the judgment rather than as a substitute for it, this method will result in a curve satisfying the eye and

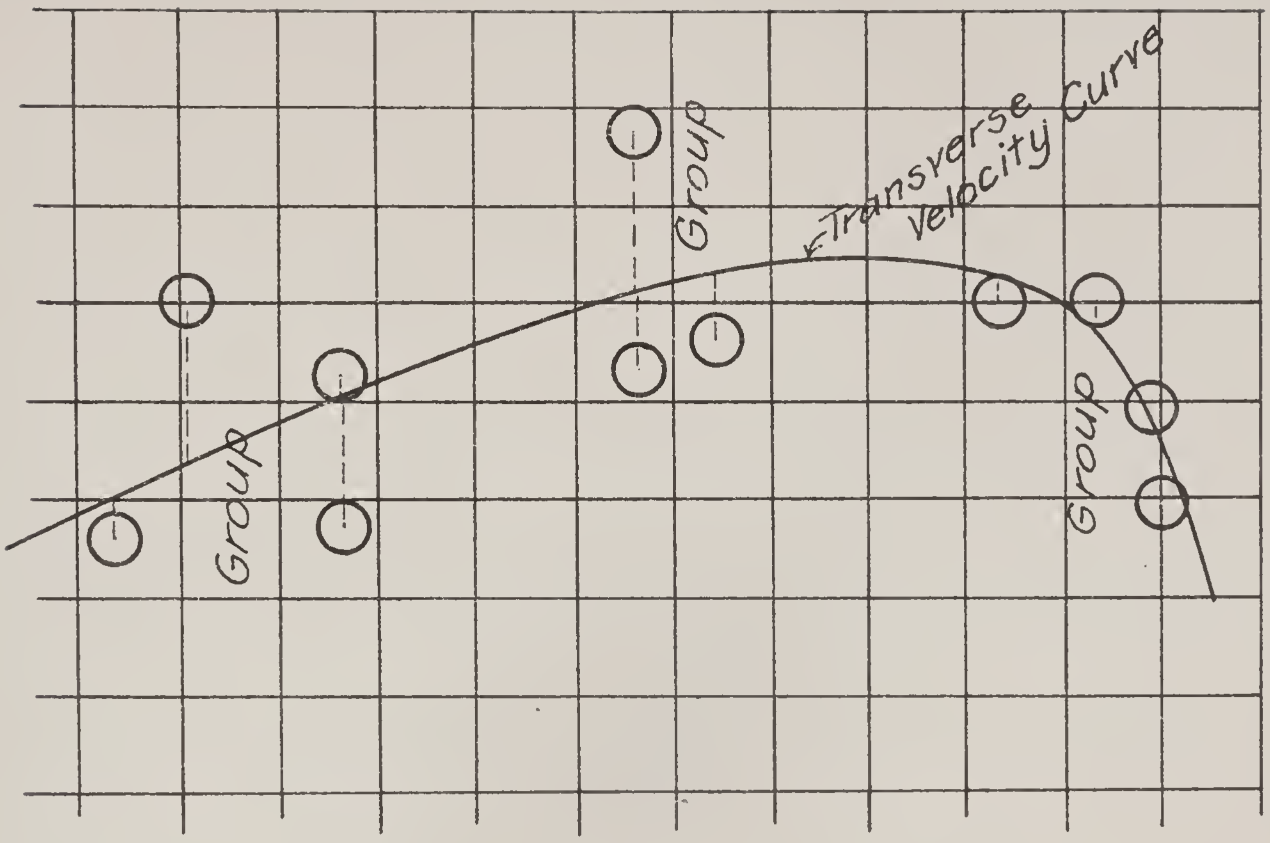


Fig. 10

giving practically the same computed discharge as any other curve passed through the same set of points somewhat differently grouped. The extreme ends of the curve must be sketched in by prolongation, and it should be remembered that the velocity usually falls away rather rapidly in the immediate vicinity of the banks.

Having thus obtained a transverse velocity curve, uniformly-spaced verticals may then be chosen as desired, their velocities scaled off, and the computation of discharge pursued as in the following article; or, if preferred, the graphical method of Harlacher (Art. 35) may be employed.

37. Procedure in Computing Discharge for Uniformly Distributed Verticals. Harlacher's method is theoretically exact, but practically the construction of his discharge curve is dependent upon the number of verticals in which depth and velocity have been measured, and is subject to inaccuracies of plotting. Moreover, a graphical method is impracticable for use in the field, where immediate determination of the discharge may be desired. Consequently it is well to notice modes of computation for which plotting is not essential.

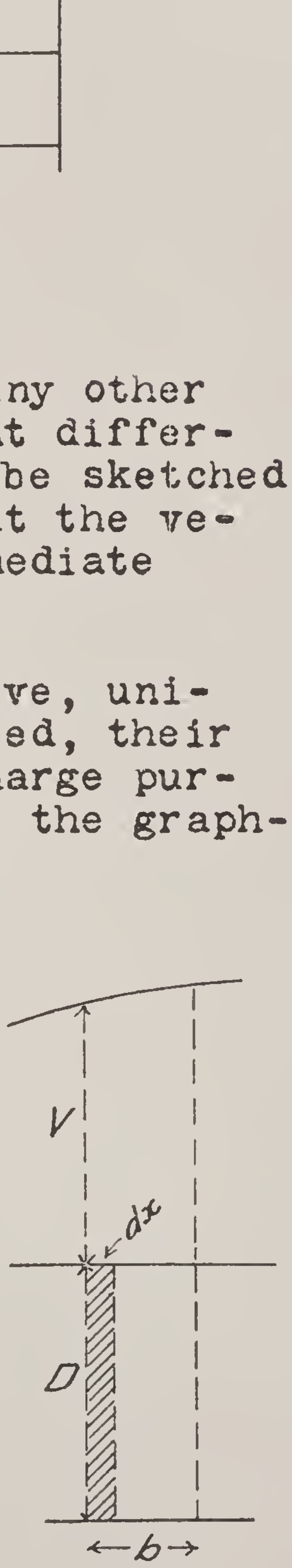


Fig. 11

Conceive the entire cross-section subdivided into vertical strips of moderate width. Let Fig. 11 show a single strip, of width b, and

suppose the bed level, thus giving a rectangle, of depth D. Past an elementary portion of this strip, of width dx, the discharge is V D dx, and past the entire strip it is

$$\int_0^b V\,D\,dx,$$

$$= D\int_0^b V\,dx$$

= D times area of corresponding portion of transverse velocity curve,

= b D times mean ordinate to said curve,

= area of strip times mean ordinate to said curve.

If the bed is not level, and the strip therefore not rectangular, the above relation does not strictly hold; but by properly limiting the width of the strips the error in the computed discharge can be kept small and a satisfactory approximation obtained.

The following detailed methods of computation, applicable to cross-sections of any shape, are now to be noticed:-

(a) (Fig. 12). Depths D, D', etc., and mean velocities V, V', etc., known for successive verticals equally spaced at the common interval b.

If we assume D and V, D' and V', etc., to represent approximately the mean depths and mean velocities for the strips, of common width b, of which the verticals are the center lines, then we may write

$$\Delta Q = b\,D\,V = \text{discharge past strip.}$$

$$Q = b\,\Sigma\,D\,V = \text{discharge past entire cross-section, possibly excepting partial strips next the banks.}$$

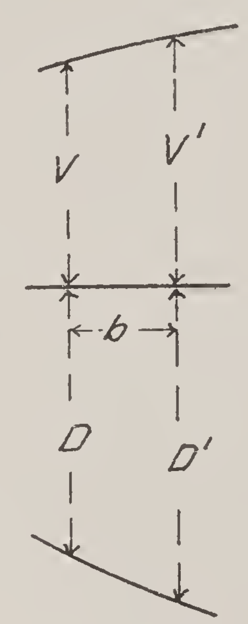


Fig. 12

(b) (Fig. 13). Depths D, D', etc., and mean velocities V, V', etc., known as before in successive verticals, equally spaced.

Taking a strip whose center line is D', and assuming V' to be the mean velocity past this strip, we have

$$\Delta Q = V' \text{ times area of strip.}$$

If, now, e f and f g be treated as straight,

$$\Delta Q = V'\,b\,\frac{D + 6\,D' + D''}{8}\,.$$

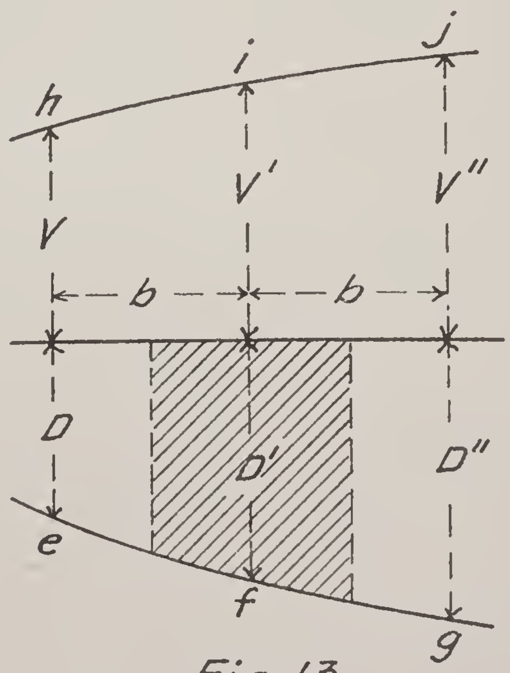


Fig. 13

(c) (Fig. 13). Taking the double strip included between D and D", and treating e f g and h i j as parabolic arcs,

$$\Delta Q = 2\,b \left(\frac{D + 4\,D' + D''}{6}\right)\left(\frac{V + 4\,V' + V''}{6}\right)$$

= discharge past the double strip.

(d) (Fig. 14). Supposing the mean velocity past the vertical center line of each strip to have been observed or scaled, then for any strip, as that between D and D', assuming e f straight, we have,

$$\Delta Q = V\,b\,\frac{D + D'}{2}\,.$$

Evidently this method may be applied to successive single strips, whether or not of equal width.

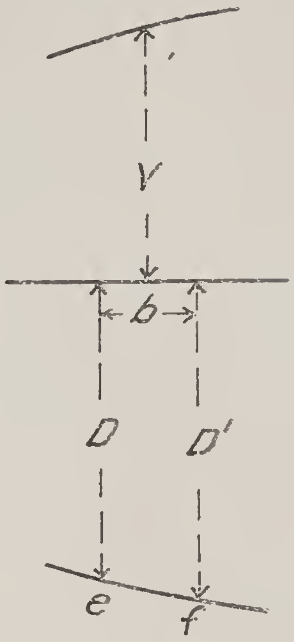


Fig. 14

(e) (Fig. 15). Supposing the velocity to have been measured at sufficient points in the vertical center line of a strip to permit laying off ordinates at proper depths and drawing the vertical velocity curve, as indicated by cross-hatching, then, treating e f as straight,

ΔQ = b times area of vertical velocity curve.

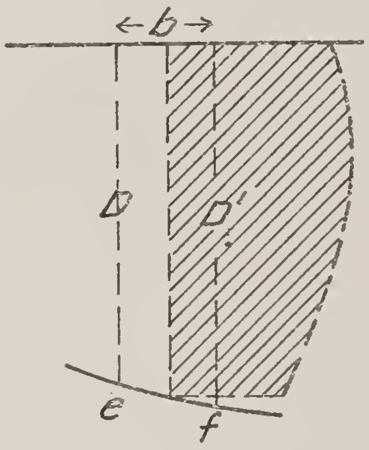


Fig. 15

It will be seen that, if soundings and mean velocities past verticals have been measured at suitable equal intervals in the cross-section, methods (a) to (d) inclusive are applicable to computing the discharge without the necessity of any plotting.

In any case, if there be an odd strip at the side of the cross-section, not covered in the above computations, the discharge past that strip must be found separately, say by method (d), and added to the amount obtained from the other strips.

38. Special Methods of Computation Allowable for Rectangular Cross-Sections. If the cross-section be rectangular, as is often true of the head-race or tail-race of a mill or power station, then,

(a) If vertical strips of equal width be taken, they will also be of equal area, and their mean velocities entitled to equal weight in averaging. As shown in the early part of Art. 37, for the discharge past any strip we may write,

ΔQ = D times area of corresponding portion of transverse velocity curve,

Q = D times area of entire transverse velocity curve, which area may be found by planimeter or otherwise.

(b) The methods of Art. 37 are evidently applicable to the special case of rectangular sections.

(c) If A represent the area of the rectangular cross-section, and the mean velocities V, V', V'', etc., have been measured past each of n verticals, numerous and pretty evenly distributed, then, closely

$$Q = A \frac{\Sigma V}{n} .$$

39. Discharge or "Station Rating" Curves. In the case of natural streams a single gaging usually has but very limited value, and where projects of importance are probable for utilizing the water of the stream, as for power or irrigation, complete knowledge is desired of the variations in daily flow through the year for a series of years. It is neither practicable nor necessary to make daily instrumental gagings for a long period of time, and resort is had to finding the relation between discharge per second and gage height at certain chosen gaging stations. Instrumental gagings are there made in at least half a dozen different stages, and often in many more, well distributed over the total range of fluctuation, and by plotting gage heights as ordinates and corresponding discharge as abscissas points are established through which a smooth curve called a discharge curve, or "station rating" curve, may be drawn. The assumption then is, that for any gage height within the range of the diagram the accompanying discharge in cubic feet per second may be read off with satisfactory accuracy.

Fig. 16 shows a discharge curve for the Susquehanna river at McCalls Ferry (Eng. News, Aug. 4, 1904), based upon 38 gagings. Such a curve having been determined, daily readings of gage height may easily be made by an unskilled observer living near by, the readings sent to the engineer and by him translated into values of discharge.

Approximately, the physical laws which hold for the discharge at any station are expressed by the formulae,

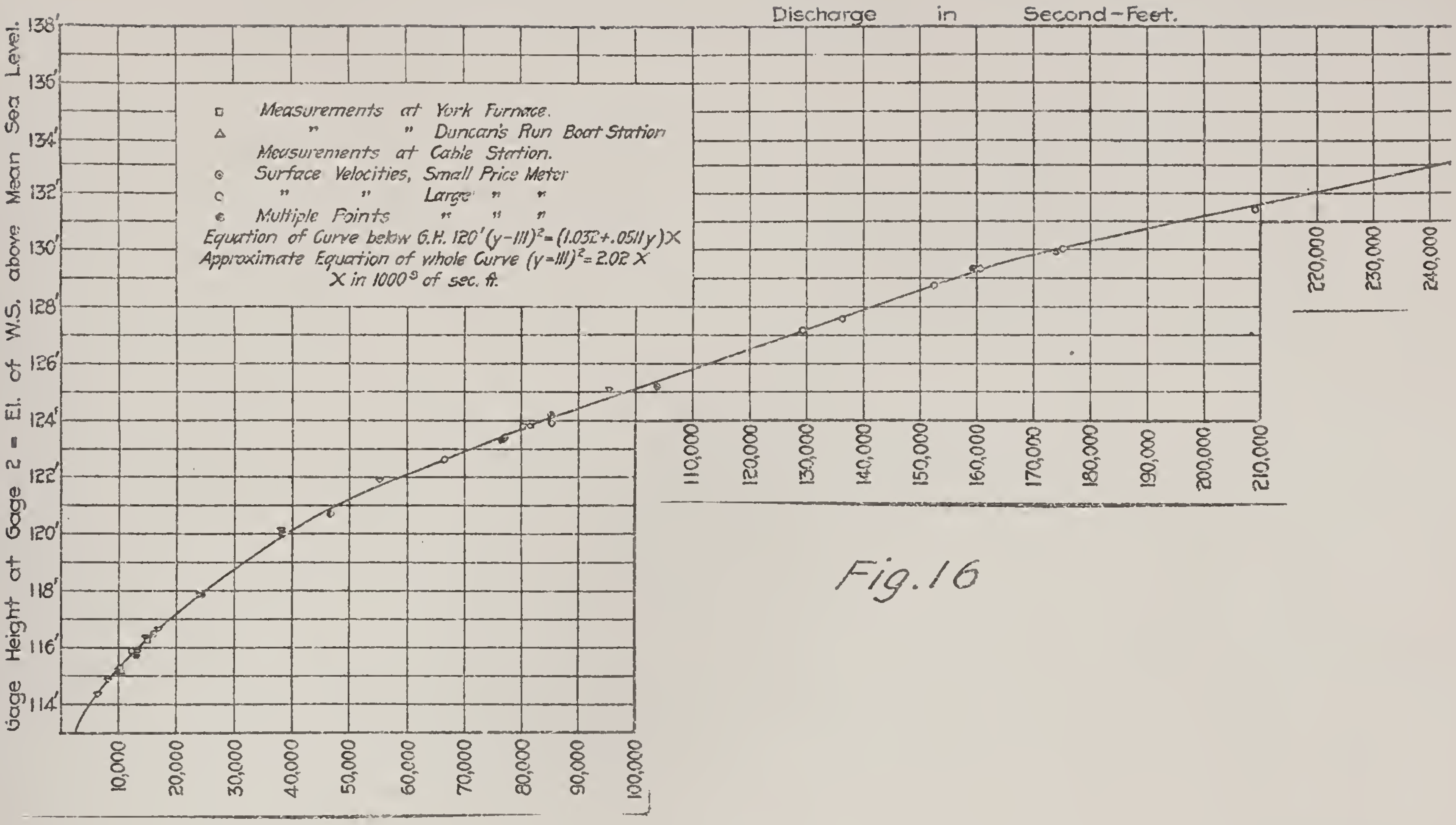


Fig. 16

$$V = C\sqrt{RS}$$

$$Q = AC\sqrt{RS},\text{ in which}$$

Q = total discharge at a given time past the cross-section, in cubic feet per second,

A = corresponding area of wetted cross-section, in square feet,

V = corresponding mean velocity past the cross-section, in feet per second,

R = ratio of area of wetted cross-section to its wetted perimeter, not including the water line exposed to the air. This ratio partly expresses the varying effect upon mean velocity past a given area of cross-section of merely varying the shape of the section, thereby varying the length of the perimeter and its relative frictional influence. The value of this ratio is called the "hydraulic radius" or "hydraulic mean depth".

S = inclination of water surface past section in question, as expressed by ratio of vertical descent in a given distance to the distance itself, the latter measured along the water surface.

C = a variable coefficient, whose value is dependent mainly upon the roughness of the channel lining, but secondarily upon the value of R, and to a minor extent

upon other conditions. Other things equal, an increase in R tends to increase C, while an increased roughness of lining tends to decrease C. This coefficient may range in value under different circumstances from 40 to 150, and even outside those limits in extreme cases.

In using a station rating curve the assumption is ordinarily made, and under suitable conditions is justified, that at a given station at all times when the water surface is at a certain height on the gage the discharge is the same,- the wetted cross-section, wetted perimeter, surface slope, roughness of lining, mean velocity and, hence, the volume flowing, being presumably unchanged. It is very important to notice, however, that this is not always true.

If the bed or banks are unstable,- subject to scour or fill, A may change so that at two different dates when Q is the same the water level and, consequently, the gage reading shall be materially different. This occurs on the Mississippi and other western rivers (Trans. Amer. Soc. Civ. Engrs., Vol. 34, p. 389), and in some of the streams of the Alps having unstable beds Tavernier notes that it is not rare for the discharge to vary in a relatively short time in the ratio of 1 to 2, or even more, for the same gage height (Annales des Ponts et Chaussees, 1907 - IV).

Again, though at two different dates when the gage height is the same A may be nominally unchanged, yet a growth of weeds in summer on the bed or banks may have altered greatly the value of C, and correspondingly of Q.

Further, if the station be within the influence of varying back water due to changing conditions down stream, such as the artificial regulation of level in a mill-pond by the use of flash-boards, or variations in pond level due to the alternate drawing down and filling up of the pond during the twenty-four hours, or ice or log jams, or rank growth of vegetation in the channel,- it is plain that A, R and S may be so changed that even for the same Q the gage height shall vary at different times.

Also it has been well established that water received in sudden and heavy freshets from the upper tributaries of a large river comes down the main channel in a "flood wave", at the front of which the slope S is abnormally large, while at the rear it is abnormally small. Consequently, at a given station on a stream, though the gage height, A, R and C, may all be substantially the same at a certain time when the level is being rapidly raised by an advancing flood and at a later time when the flood is subsiding, yet Q may in the first case greatly exceed its value in the second. This is clearly shown in the discharge curve of the river Fisza (Fig. 17). In one or two exceptional cases Ellis found the reverse of this law to hold true at a station on the Connecticut river (House Ex. Doc. No. 101, 45th Congress, 2d session).

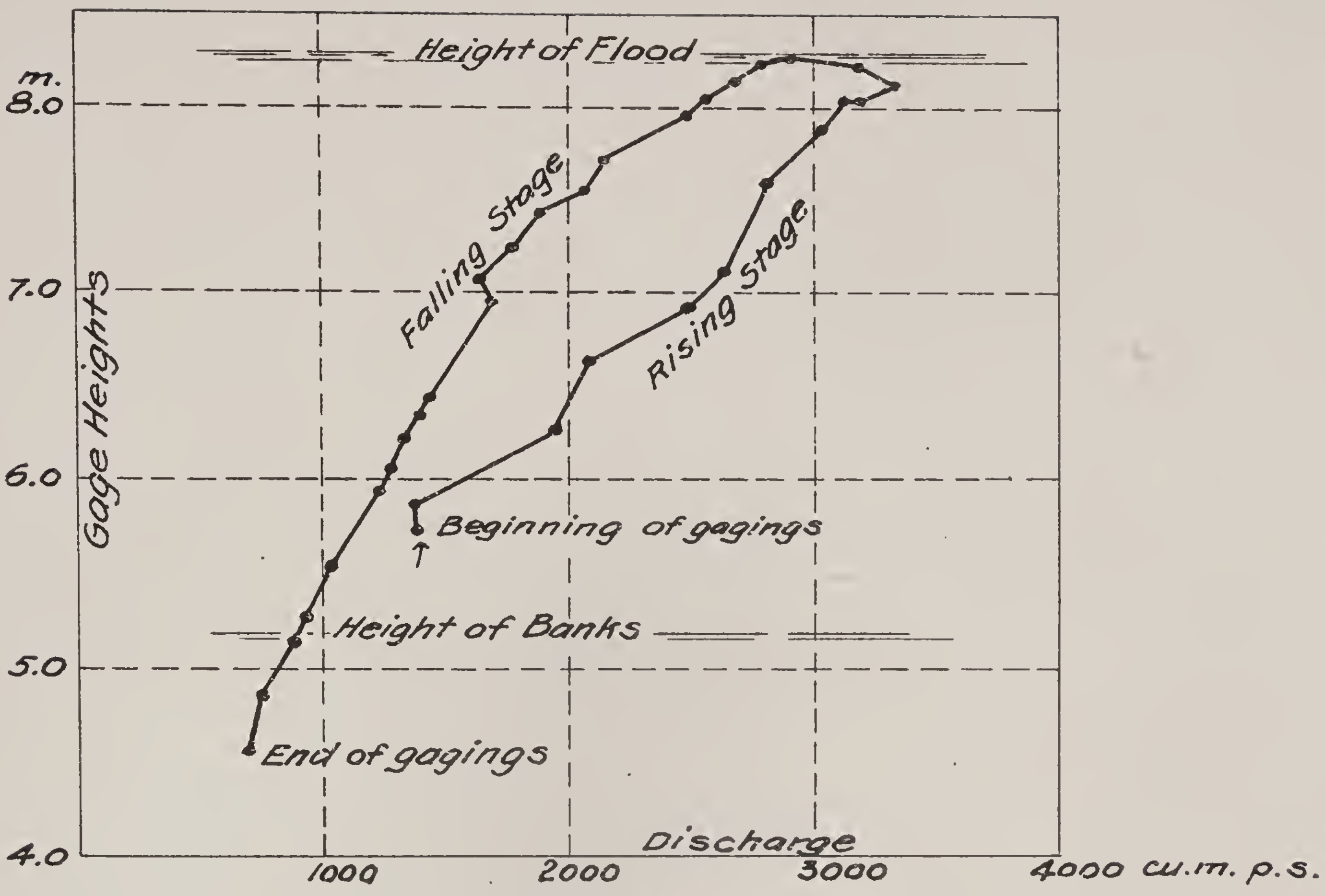


Fig. 17

It is clear, then, that much care and caution need to be exercised in establishing a permanent gaging station, and in interpreting the observations for gage height as measures of discharge.

The normal shape of the discharge curve is considered to be a parabola, convex to the axis of gage heights, and tangent to that axis at the level of zero flow, which may or may not be that of the lowest point in the cross-section. For high stages the curve usually becomes approximately straight.

While a half dozen points may, under favorable circumstances, suffice to determine this curve satisfactorily, more are needed for low stages than for high, because of the sharper curvature as the tangent point is approached.

In plotting the curve, scales should be chosen that will give good intersections of the curve with lines parallel to the axes, and that will permit of plotting and of reading off values with due precision.

Since it is quite unlikely that a smooth curve can be passed through all the plotted points, it must be averaged among them as well as possible. This is sometimes done by assuming a general equation such as $Q = A + Bx + Cx^2$ (x representing gage height), and finding by the method of least squares the most probable values for A, B and C for the given

observations, all these having equal weight. More often, and sufficiently well in general, the curve is sketched in by eye, points being weighted according to any special knowledge that may be had as to their probable accuracy. According to the practice of the U. S. Geological Survey, a point obtained from any single gaging that varies from a well defined curve given by other gagings by more than from 5 to 8% (accordingly as the conditions at the station are favorable to good results or not) is considered either to be in error or to have been affected by a change in channel (Water Supply and Irrigation Paper, No. 94, p. 22).

40. Discharge Tables. A discharge curve having been drawn, it is usually thought preferable to scale off values and put them in tabulated form for subsequent use, rather than refer repeatedly to the diagram. The discharges corresponding to successive heights on the gage at intervals of, say, tenths of a foot are therefore to be read off from the curve, with such refinement only as seems warranted by the general accuracy of the gagings; and in order to make the table consistent in itself the scaled values are corrected, if necessary, by taking first, and sometimes second, differences and adjusting them so that they either remain constant, or increase with increasing gage height, with fair regularity, as seen in the following illustration:-

Gage Height in Ft.	Discharge in C.F.P.S.	1st Difference
4.90	1390	
		60
4.95	1450	
		60
5.00	1510	
		65
5.05	1575	
		65
5.10	1640	

41. Extension of Discharge Curve beyond Limits of Observations. In so far as points have been fixed by instrumental gagings, the discharge curve can be drawn with confidence; but it is frequently desirable to extend it to high or low stages in advance of opportunity for actual gagings at such stages. Such extensionsneed to be made with much caution, or serious errors may result in the estimated discharge. The following methods are available:-

(a) Extend curve by eye beyond limits of observations. This will give a rough approximation, but is not the most reliable method. In the lower portion the curvature changes rapidly, and a safer plan for extending the curve downward is to assume the lower portion a parabola and plot it by coordinates laid off to logarithmic scale instead of to natural scale. This will change the curve

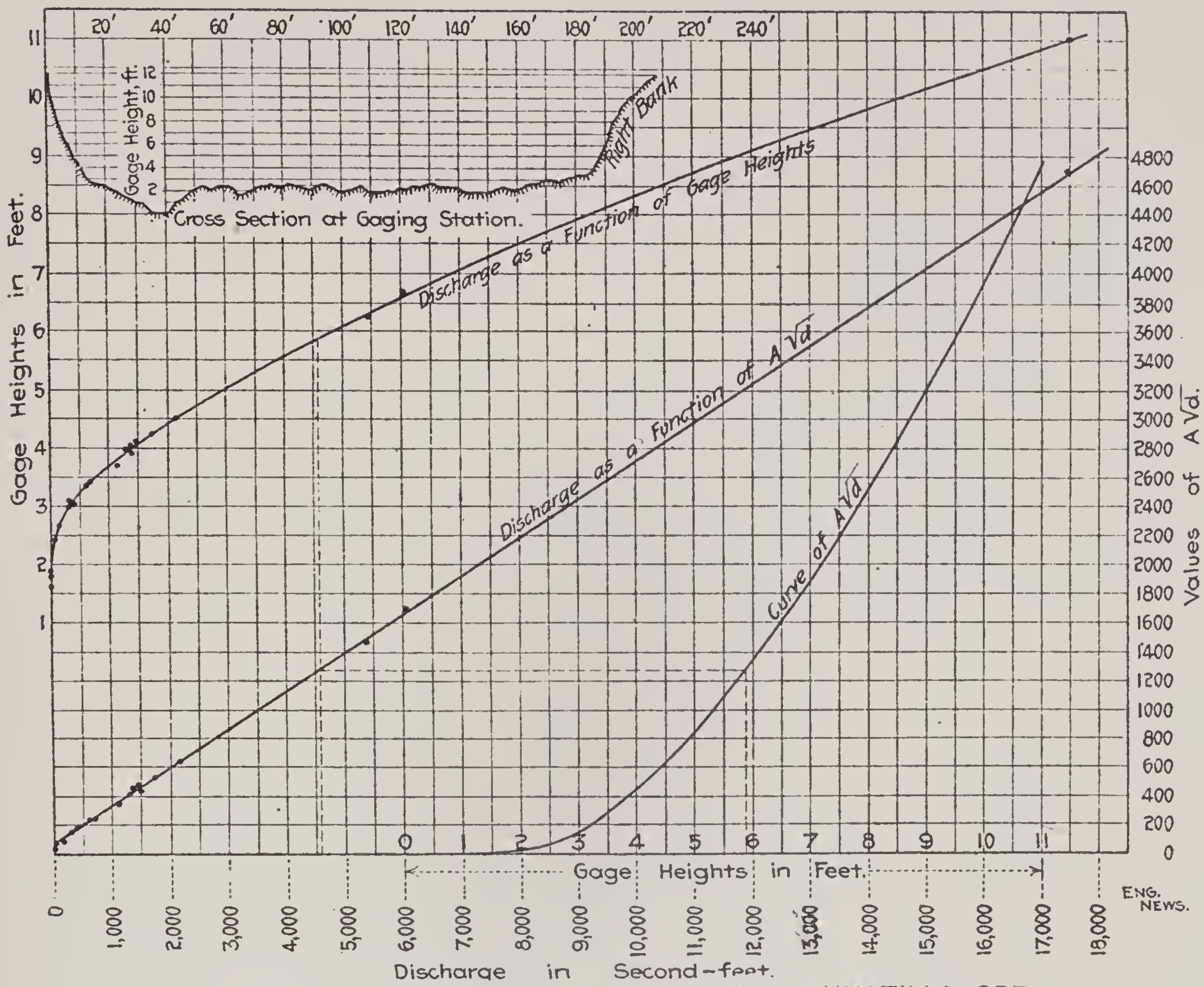
to a straight line, which may then be prolonged downward with considerable accuracy.

The upper portion of the discharge curve is apt to be approximately straight, but unless its direction is well determined by known points its extension may lead to wide error.

(b) The second method takes account of the relation that at all stages $Q = A\,V$. The values of A can be found with any desired accuracy for all stages of flow from an area curve based upon soundings and cross-sections of the banks. The curve for V usually has but slight curvature (normally concave to the axis of gage heights) and can be extended <u>upward</u> with more accuracy than the discharge curve can be independently extended. This done, the products of corresponding values of A and V, scaled from their respective curves, will give additional values for fixing the extension of the discharge curve. This method is unsafe, however, for an extension of the curve <u>downward</u> into low stages, since there the curvature of the curve of velocity changes more rapidly than at high stages, even reversing under some conditions, and the proper extension of the curve is uncertain.

(c) An extension of the discharge curve <u>upward</u> from medium to high stages may be based upon the assumption that within that range of heights C and S in the formula $Q = A\,C\,\sqrt{R\,S}$ remain approximately constant. We may then write $Q = K\,A\sqrt{R}$, or, since d, the mean water depth, $\left(\frac{\text{area}}{\text{surface width}}\right)$, which is easily determined, varies but slightly in a natural channel from R, we may write $Q = K\,A\,\sqrt{d}$, for which equation the graph will be a straight line and may readily be extended (see Fig. 18, taken from article by J. C. Stevens in Eng. News, July 18, 1907). While this method has given good results in particular cases, it must be applied with caution, since C and S are not strictly constant, and may even vary greatly between different stages, especially between low and high water.

42. <u>Area, Mean Velocity and Discharge Curves: Theoretical Considerations</u>. Reason has been shown why gaging station curves are sometimes required separately for area of cross-section, mean velocity past the cross-section, and discharge. It is true that the drawing of these consists mainly in the simple sketching of smooth curves through plotted points. These points are bound to be more or less in error, however, so that no smooth curve perfectly joins them, and the observations upon which they are based are time-consuming and expensive. Now the curves possess certain general properties, a knowledge of which may often serve to limit the number of observations, or to assist in interpreting them or in testing



DISCHARGE CURVES FOR UMATILLA RIVER AT UMATILLA, ORE.

Fig. 18.

their accuracy. It is worth while, therefore, to examine briefly the underlying theory, which is set forth in more detail by Tavernier in Annales des Ponts et Chaussees, 1907 - IV.

Area Curve. For every computation of discharge based upon an instrumental gaging the corresponding area of cross-section must be known, either directly from soundings made at the time of the gaging, or from soundings made at some other time and corrected for the difference in water level. There are, then, likely to be at least as many instrumental determinations of area as of discharge, and preferably the profile of bed and banks should be closely determined up to the level of extreme high water. The area can then be computed for any stage, and by plotting a suitable number of points an area curve can be drawn, as already noticed, showing the relation between gage height and area. Such a curve tends to reveal gross errors in measurement or computation of the individual areas upon which it is based; serves as a standard to which subsequent determinations of area may be referred as a test of their probable accuracy, or for revealing the occurrence of scour or fill in the channel; and is of aid in drawing, and especially in prolonging, the discharge curve.

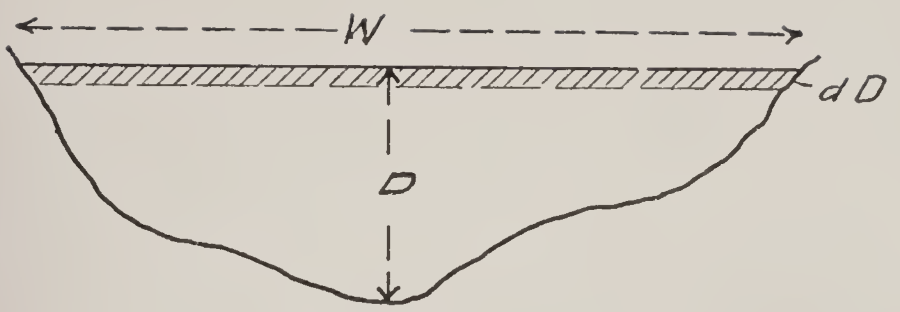


Fig. 19

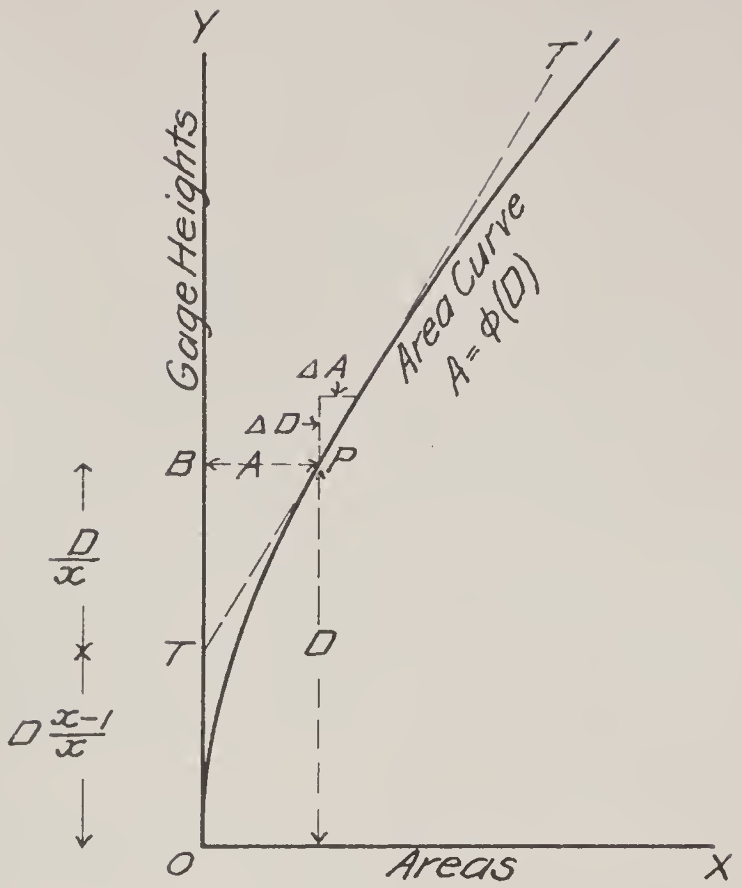


Fig. 20

Let Fig. 19 represent a stream cross-section, and Fig. 20 the corresponding area curve.

Let A = area of wetted cross-section at any stage,
W = corresponding surface width of stream,
D = corresponding water depth above lowest point of bed (equals also gage height when zero of gage corresponds to lowest point of bed).

Considering $A = \phi(D)$, from any differential strip of thickness d D and area d A it is seen that $W = \frac{dA}{dD}$. Hence, for any position of water surface, the surface width is equal to the first derivative of the area with respect to the depth, that is to say, to the fraction which expresses the corresponding inclination of the curve $A = \phi(D)$ to the axis of gage heights. Thus in Fig. 20, at any point P the surface width W is equal to the inclination of the tangent T T' to the Y axis, the coordinate D representing the water depth (gage height) and A the corresponding area. For a rectangular cross-section, in which case the banks will be vertical, W will be constant, and the curve $A = \phi(D)$ will be a straight line. In rare cases the banks may overhang; but in general they will be inclined away from the vertical, with a more or less gentle slope, W will increase continuously with the depth, and the curve $A = \phi(D)$ will be convex toward the axis of gage heights. It will be tangent to that axis at zero depth, unless the cross-section have a level bottom, in which case the curve will meet the axis at an inclination equal to W.

The actual inclination on the paper plot of an area curve, at any point, will of course vary with the relation between the scales adopted for depths and areas, respectively. Therefore it is necessary to notice that in practically applying the relation $W = \frac{\Delta A}{\Delta D}$ to finding the slope of a tangent at any point, as P (Fig. 20), consistent scales must be used. If ΔD be actually one foot, in space, then ΔA will contain the same number of square feet that W does of linear feet; hence, if ΔD be laid off as one foot to the scale of depths, ΔA must be laid off equal to W units <u>to the scale of areas</u>.

Assuming the area curve approximately parabolic (for straight slopes it would be strictly so), we may write,

$A = K D^x$, K being a constant, and x an exponent varying with the slope of the banks.
$x = 1$ for vertical banks,
x lies between 1 and 2 for banks concave upward,
$x = 2$ for straight, sloping banks,
$x > 2$ for banks convex upward.

Evidently B T (Fig. 20) $= \frac{D}{x}$. In other words, the tangent at P meets the axis of gage heights at a distance O T from the origin equal to $D \frac{x - 1}{x}$. The value of D being known, and O T measured on the plot, x can be found, then K, and the equation of the corresponding portion of the curve is completely determined, although the values of x and K may vary for different portions.

<u>Mean Velocity Curve</u>. For each gaging, Q and A becoming known, V, the mean velocity past the entire cross-section, follows from the relation between Q and A, and by plotting a series of values of V for different gage heights a curve may be determined. As elsewhere stated, this curve is likely to be concave toward the axis of gage heights, becoming nearly straight for high stages provided the stream does not overflow its banks.

Let us apply the formula $V = C\sqrt{R S}$, and let d represent the average water depth in the cross-section, $= \frac{A}{W}$. Where the depth is small compared with the width, as is often the case in natural channels, there is no great error in assuming $R = d$.

We may then write $V = C S^{\frac{1}{2}} d^{\frac{1}{2}}$. But d being a function of D, we may further write as approximately true for those cases in which C and S are fairly constant $V = C' D^{\frac{1}{2}}$, in which C' is

assumed constant. The graph of such a curve is illustrated in Fig. 21. The first derivative of V with respect to D shows the inclination of the tangent to the curve at any point P, with respect to the axis of gage heights, to be $\frac{C'}{2D}$. It will also be seen that the tangent meets the axis of velocities at a distance from the origin equal to $\frac{V}{2}$, and the axis of gage heights at a distance (-D). Under the assumptions here made the curve should be tangent at the origin to the axis of velocities, except in case of a bar below the gaging station causing dead water in low stages, when the velocity tends to disappear at some point above zero depth. In such case the curve is said sometimes to show a reversal, and to become convex to the axis of gage heights on approaching it.

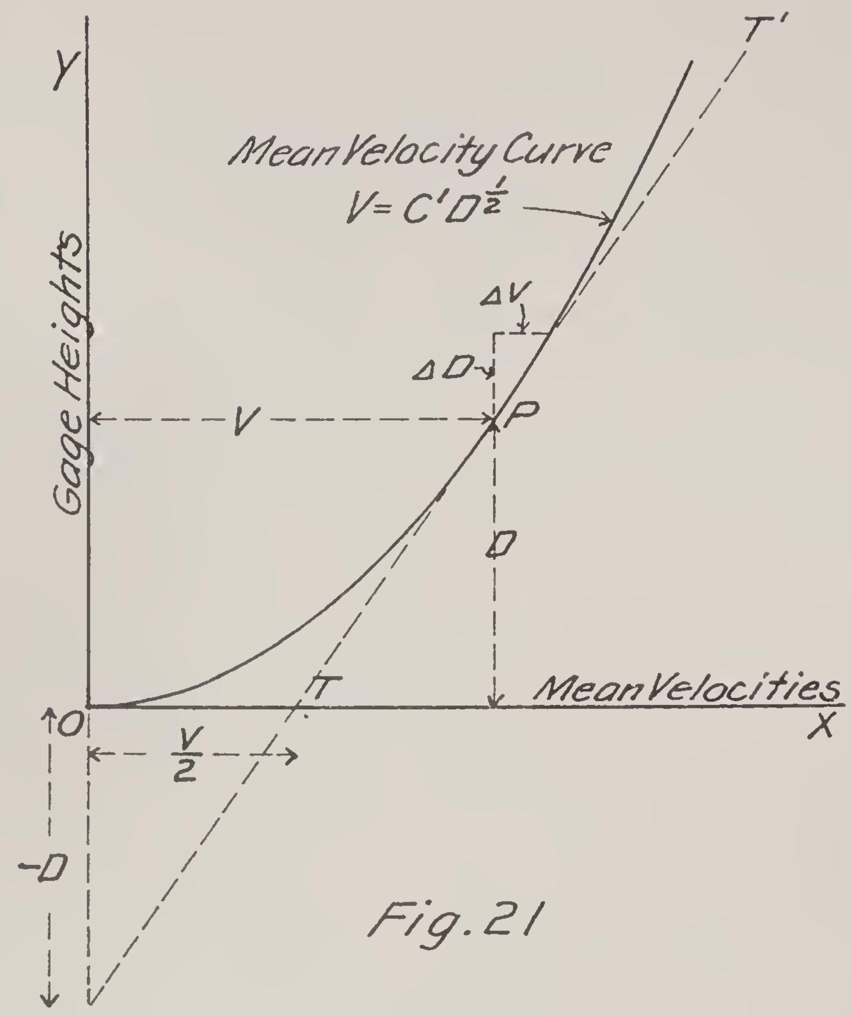


Fig. 21

Discharge Curve. Continuing the assumptions already made, and combining the expressions for A and V, we have

$Q = K C' D^{\frac{2x+1}{2}}$, the graph of which is represented in Fig. 22. It will be seen that a tangent to this curve at any point P should meet the axis of gage heights at a distance from the origin equal to

$\left(\frac{2x - 1}{2x + 1}\right)$ D. As previously

noticed, the curve is normally convex to that axis, and ordinarily tangent to it at the origin; but, like the mean velocity curve, it will meet the axis at a height above zero in the case of dead water being caused at the gaging station by a bar.

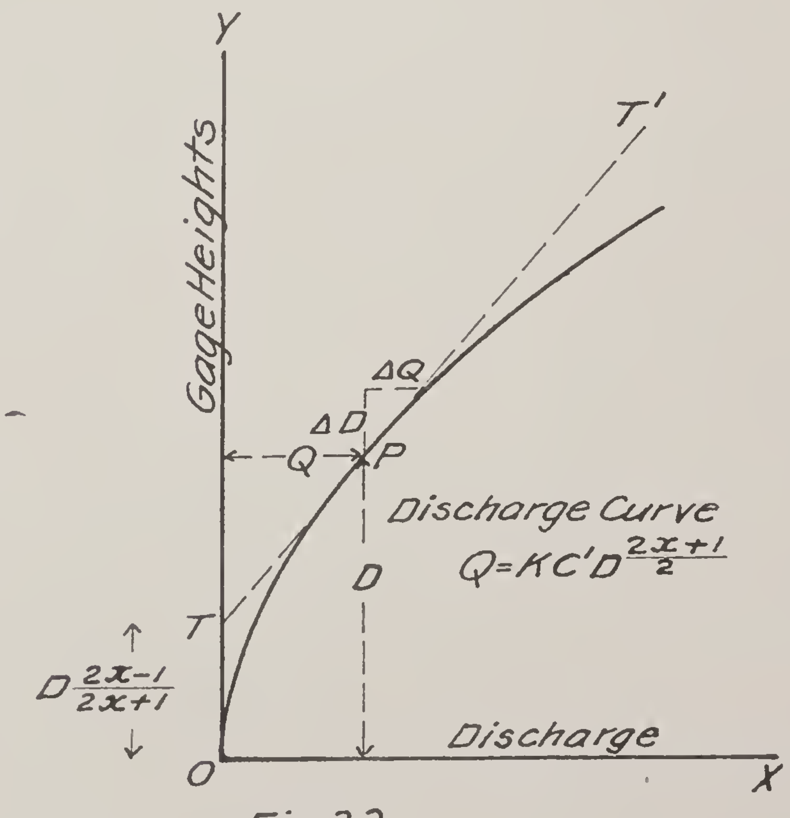


Fig. 22

43. Use of Discharge Curves for Streams of Unstable Bed. Attention has already been called to the fact (Art. 39) that, if bed or banks be unstable, the constancy of the relation ordinarily assumed between gage height and discharge does not hold. Tavernier even says that in many cases it has been found for such streams in the Alps that the gage height is much more influenced by the rising or lowering of the bed than by increase or decrease of discharge (see article on Gaging of Streams with Unstable Bed; Annales des Ponts et Chaussees, 1907 - IV). Special caution must therefore be used in applying discharge curves to such streams. Instrumental gagings must be made at relatively short intervals during periods of shifting bottom, and some method of interpolation be employed for estimating from gage heights the probable values for discharge on intervening days. Two such methods are described by Hoyt and Grover (River Discharge, pp. 95 et seq.), and are substantially as follows:-

Stout Method. Plotting gage heights as ordinates, and instrumentally-measured discharges as abscissas, those resulting points, especially if chronologically consecutive, which may be joined by a discharge curve of reasonable shape, are so joined, thus establishing an approximate standard curve, such as Curve A of Fig.23, in which points given by gagings of May 1st, 11th and 21st are joined. If now on June 5th, at an observed gage height of 13.8, an instrumental gaging gives Q = 2300 cubic feet per second, we may get that value from the standard curve by correcting the gage by + 1 foot. And if on June 20th, at an observed gage height of 11.3, a gaging gives Q = 1150 cubic feet per second, we may get this value from the standard curve by correcting the gage height by - 1.2 foot. By plotting a series of such corrections as ordinates, and the corresponding time intervals as abscissas, joining the plotted points by a continuous curve, as shown, and assuming the curve to show the probable law of change of corrections, we

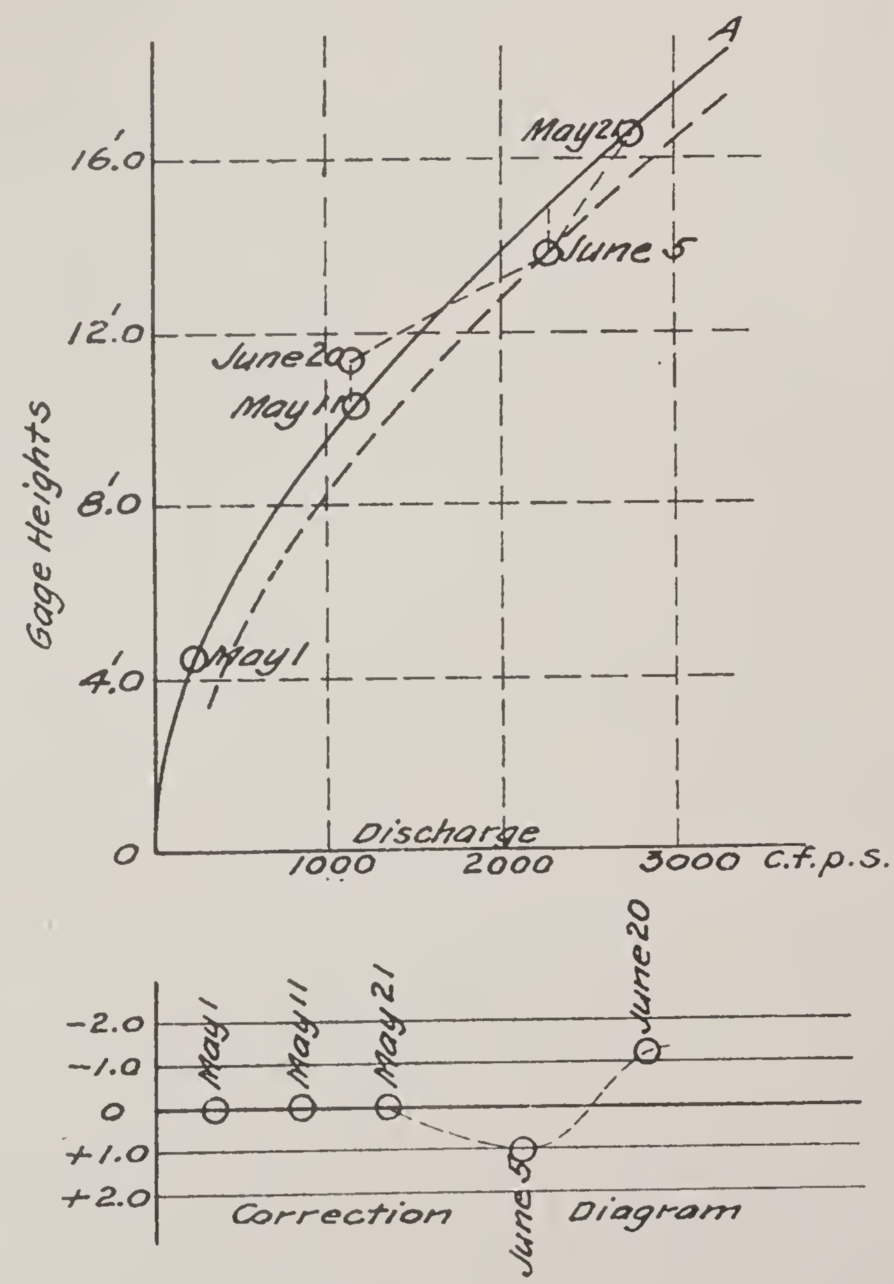


Fig.23.

may scale off for any intermediate date the correction to be applied to the observed gage height for that day. With the corrected gage height we may then read from the standard discharge curve the probable discharge for the day in question.

Bolster Method. A standard curve is constructed from time to time as accumulated gagings seem to justify. If curve A, Fig. 23, represent the standard for present use, and a reliable gaging on June 5th give the point shown for that date, then the standard curve is shifted bodily downward, parallel to its original position, until it pass through the point mentioned, as shown by dotted line. In its new position the curve is assumed to show the true relation between gage heights and discharge for the channel as it then exists. For any date intermediate between May 21st and June 5th the curve may be moved to the corresponding interpolated position, and the probable discharge for the known gage height read directly from it in that position. The necessity for this shifting of the discharge curve up or down for streams of unstable bed, if it is to show correctly at different times the relation between gage height and discharge, has been plainly noticed and called to attention in connection with Mississippi river measurements (see Trans. Amer. Soc. Civil Engrs., Vol. 34, p. 449).

44. Methods Available for Measuring Velocity. Although many devices are possible, and many have been tried, for measuring velocities in open channels, by far the most of present-day stream gagings are made with the current meter, having a wheel which is turned by the force of the current. If properly rated, and intelligently used, under reasonable conditions, this will give in general greater accuracy than other devices, and will give them more quickly and economically.

Very extensive and valuable series of measurements have been made, however, with floats; and for rectangular flumes it is probable that no instrument is superior in accuracy to the rod or tube float. For sluggish currents, also, in which the meter wheel would either not turn at all, or would give uncertain results; and in streams with many floating weeds or much floating debris, which might clog or injure a meter, some form of float is in order.

Modifications of the Pitot tube have been employed somewhat by French engineers. The method has great value for finding the velocity in pipes flowing under pressure, but is of limited service in open channels, although it may be useful in small, clear streams, especially if the velocity is exceptionally high.

The formula $V = C\sqrt{R\,S}$ is not infrequently applied to stream measurements where direct gagings are impracticable, but the results are often subject to large error.

About 1840, large paddle wheels, between 15 and 20 feet in diameter, were used for gaging the flow in some of the Lowell canals, and are referred to by Francis in "Lowell Hydraulic Experiments".

Rarely, some kind of coloring matter has been added to a small stream, such as that in a sewer, the color permitting the eye to note the moment of passage by the upper and lower extremities of a section of known length, as in the case of floats.

The plan has been slightly experimented with, also, of mixing a suitable chemical in known volume with a small stream, and measuring the resulting dilution as an index of the volume flowing.

Observation of the respective temperatures of two small streams immediately above their junction, and of the combined stream below the junction, has been made the basis of determining their volumes.

Sundry other very exceptional means of measurement are referred to on p. 17 of Water Supply and Irrigation Paper, No. 95, of the U. S. Geological Survey.

45. Advantages and Disadvantages of Floats. Floats are made either single and shallow, to be used at the surface; or as a combination of a small surface float and a larger and principal sub-surface float, joined by a cord; or in the form of a weighted tube or rod, to reach from the stream surface as nearly as may be to the bed. For general purposes of measurement they present the following advantages:-

(a) Interfere but little with the natural motion of the water, even in small streams.

(b) Measure the velocity directly, and unperceived large errors not likely.

(c) Can be used in streams of any size, and at all velocities.

(d) Little affected by silt or weeds.

(e) Give desired forward velocity, i.e., the component of actual velocity at right angles to the section.

(f) Cheap and easily repaired.

On the other hand, the following disadvantages are to be noticed:-

(a) Difficult to regulate their course.

(b) Impelled by adjacent moving mass of water only, velocity of which may be either maximum or minimum of the varying impulses due to pulsations. Numerous observations therefore necessary for accurate results. Cunningham, from his experiments in Ganges canal, considered about 50 repetitions of float runs needed.

(c) If bed is irregular, no form of float can be run close to bottom throughout its course.

(d) At least three men needed for field work, and on large streams additional men for boats.

(e) Several cross-sections may have to be measured to determine the mean for the float course.

(f) Influence of surface float and cord (in case of double floats) upon velocity of lower float. In great depths exposed surface of cord may exceed that of float; thus in some of Humphreys and Abbot's experiments on the Mississippi the cord presented an area 50% greater than the lower float.

It will be seen that in determining vertical velocity curves by double floats the influence of the surface float tends to carry the observation curve wholly outside the true curve if the maximum velocity be at the surface; but if the maximum velocity be below the surface the observation curve tends to fall inside the true curve down to the depth where the velocity equals the surface velocity, and thence downward to fall outside the true curve.

(g) Uncertainty as to the relative positions of surface and sub-surface float.

(h) Tipping of sub-surface float, changing its exposed surface and increasing its liability to be carried upward in eddies.

46. <u>Surface Floats</u>. Surface floats are likely to be used but seldom, except in a hasty reconnoissance, or in streams swollen by flood and dangerous for more exact methods of gaging. They necessarily have some degree of submergence, but this should be slight, and there should be as little as possible exposure above the water, so as to reduce to a minimum air resistance and the effect of wind currents. In small streams it is desirable that the float should itself be relatively small. If it be circular it will have the advantage of presenting in all positions the same area to the current. Very simple objects may be used; in small streams an orange answers well the above requirements, and its color permits the eye easily to follow it. In large rivers a flag may be needed to mark the float. In streams made unsafe for boats by floating debris or

ice, pieces of drift or cakes of ice may themselves answer as a substitute for artificial floats.

47. Double Floats. Double floats consist of a relatively small surface float and a larger sub-surface float joined to the former by a cord and adjusted to run at the desired depth. The surface float permits of observing the course of the main float and of recovering it at the end of the run. Ellis has pointed out the features mentioned below as important in a good double float, and they will be seen to hold in the form used by him in extensive gagings of the Connecticut river at Thompsonville in 1874 and illustrated in Fig. 24:-

(a) The lower float should offer large lateral resistance and this should be equal in all directions, on account of rotation.

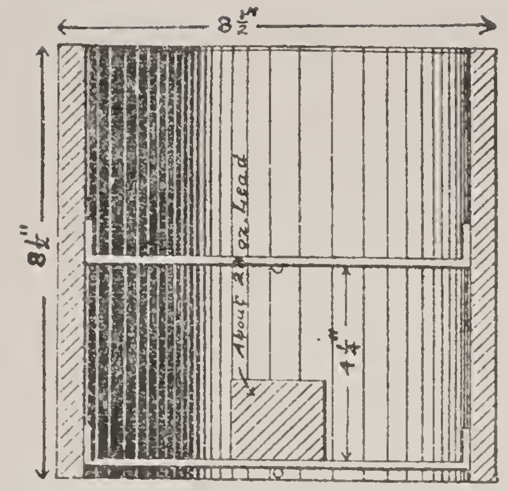
(b) It should offer small vertical resistance, so as not to be affected by eddies or downward currents.

(c) It should have sufficient stability of flotation to stand upright in the water.

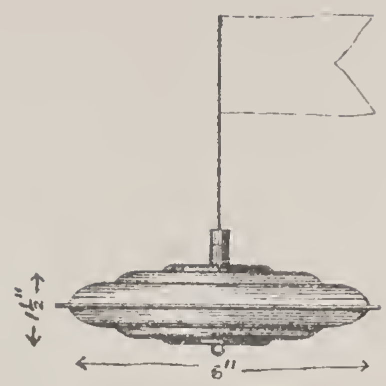
(d) It should have sufficient preponderance of weight to prevent floating upward in eddies and to keep the connecting cord vertical, but not such as unduly to increase the size of the surface float.

(e) The surface float should be as light and as small as practicable and yet sustain the required weight, and its form should be such as to offer minimum resistance to the water and to wind.

(f) The connecting cord should be the smallest that is consistent with requisite strength.



Section of Sub-Surface Float.



Side view of Surface Float

Top view of Sub-Surface Float.

Top view of Surface Float

Manner of using Floats.

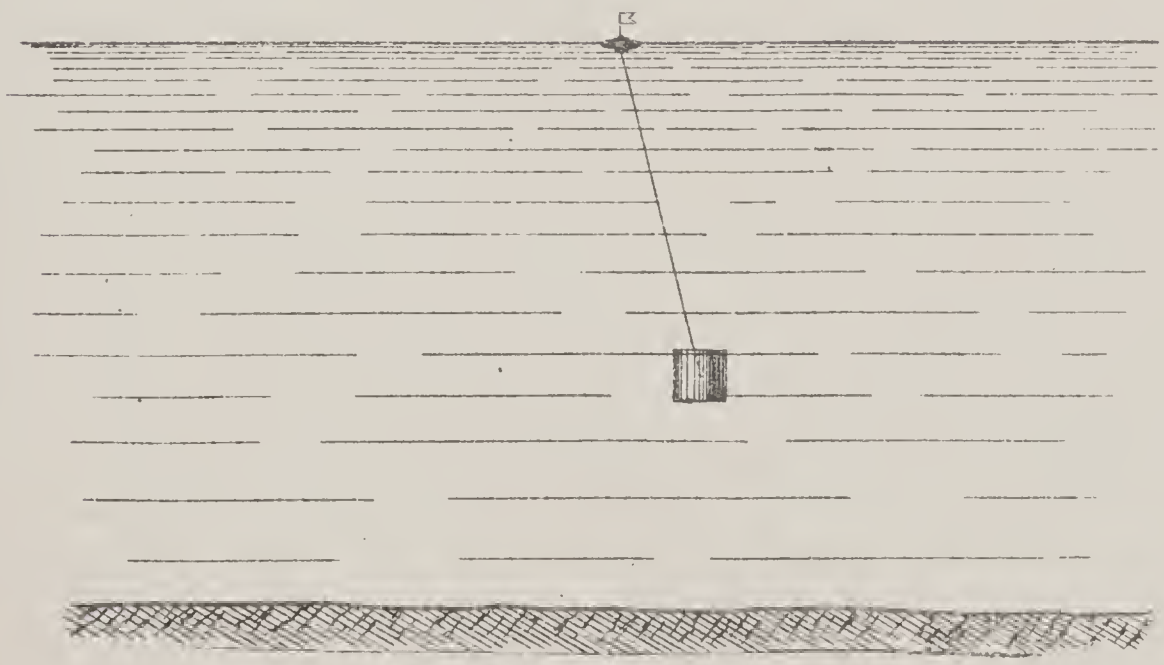


Fig. 24

48. Rod or Tube Floats. These consist of wooden rods or metal tubes, weighted at the bottom so as to float with axis vertical. They should project slightly above the water surface, and should extend downward as nearly as practicable to the bed, preferably nine-tenths or more of the full water depth. For a less relative immersion than nine-tenths the relation is not so well known, between the observed velocity of the float and the mean velocity of the water for the entire depth. Such floats are simple in construction, are free from the uncertainties attending the use of double floats, give directly a close approximation to the mean velocity past a vertical, and for channels with smooth bed and not of excessive depth afford a very accurate means of finding the discharge.

In Mississippi river gagings rod floats as long as 35 or 40 feet have been used, but greater lengths cannot be successfully handled. This type is especially well suited to artificial canals, and for over fifty years tubes have been the standard device for gagings in the canals at Lowell. Those introduced by Francis were 2 inches in diameter, made of tin plates soldered together and weighted with a solid cylinder of lead at the bottom sufficient to sink the tube nearly to the desired depth. By dropping in a little additional weight the tube could be immersed to the exact depth required, leaving about four inches of length above the surface. The top was closed with a cork. Latterly brass tubes have been adopted for some of the canals.

Cunningham used 1-inch tin tubes in the Ganges canal gagings, while in certain experiments for determining the rate of travel of fresh water down the Thames logs about a foot in diameter and about 12 feet long, weighted with iron at the end, have been employed.

49. Details of Float Measurements.

(a) The float should be set free sufficiently far above the upper range line to allow it to attain its normal position and velocity before reaching the line.

(b) The length of run from upper to lower range lines should be no longer than is necessary properly to limit errors of observation. The common range in length is from 50 to 200 feet, varying with the speed of the current and somewhat with the size of the stream, but in a very slow current a much shorter distance might suffice.

(c) The points of crossing the upper and lower range lines are commonly fixed by transit intersections from the shore in the case of large streams; and by graduated lines or beams spanning the channel in the case of small streams or canals.

(d) The timing may be done by a stop watch in the hands of an observer, or by a chronometer and chronograph with electric connections to observers at the upper and lower transit lines, or by a similar device with electric connections and automatic devices operated by the float itself as it enters upon and leaves its measured course (see Report of Chief Engineers, U.S.A. 1883, p. 2226).

(e) Sub-surface floats are frequently run in series so as to obtain in each set velocities in a vertical plane at successive intervals of depth, varying from one foot, for rivers of moderate depth, to 5 feet for deep rivers like the lower Mississippi.

(f) Field notes should record: names of observers; name of stream; locality of gaging, with sketch of site; date of gaging; time of beginning and of ending gaging; kind of float used; gage height at suitable intervals for determining mean value; reference of zero of gage to permanent bench mark; test of watch for error; remarks as to wind, causes of abnormal runs, etc.; and for each run, the depth of immersion of float, distance out from reference line to crossing by float of upper and of lower range; transit angles, if used for locating float; elapsed time by watch in seconds and tenths. Soundings, angles or other readings for locating them, and accompanying gage heights, also form part of the complete record.

50. The Current Meter. This instrument has a wheel furnished with vanes or cups, made to revolve by the current, the number of revolutions being registered upon a suitable counter by means of gearing or by electrical connections, or else indicated by sound. The relation between the number of revolutions per unit of time and the velocity of the current is determined by experimental rating. Because of its convenience and the accuracy that may be attained with it under a great variety of conditions, the current meter is in general preferable to any other device for measuring velocities in open channels. The following advantages are also to be noticed:-

(a) Observations are confined to one section, and there is less need than with floats of a straight reach of channel of uniform dimensions.

(b) Observations can usually be taken more rapidly than with floats, a smaller party is required for the field work, and computations of discharge are shorter.

(c) Variations of velocity are averaged during the time of an observation.

(d) Observations can be taken tolerably close to the bed and banks.

(e) The instrument admits of the easy and rapid measurement of the mean velocity past a vertical, and of the particular velocity at any point.

(f) At dangerous sites, by the use of cables and travelers, it is possible for one man to do the entire work of gaging from the shore.

Nevertheless, the current meter is a somewhat delicate instrument, and success in its use depends upon the selection of a suitable type for the special work in hand, upon its mechanical perfection, its condition when used, its proper manipulation during gaging, and the accuracy of its rating. Certain inherent defects, more or less serious according to the style of meter and the conditions of its use, as enumerated by Ellis, are:-

(a) Difficulty of rating and liability to change of rate if lubrication be varied, or bearings become worn, or the instrument be injured.

(b) Irregular or suspended action at low velocities.

(c) The meter may register full revolutions only, which may be of several feet pitch, in which case the error of a partial revolution may be of importance.

(d) The action of the meter is liable to be impeded by floating debris, and to be irregular in water containing sediment.

(e) If free to take its own direction, the meter is likely to stand at a slight angle from the current; and also if the current is oblique to the cross-section the meter will not give the desired component of velocity at right angles to the section.

(f) Faults of construction, and liability to derangement of mechanism.

In its primary form the current meter was invented about a century ago by Woltmann, a hydraulic engineer of Hamburg. His meter had to be removed from the water for each reading of the number of revolutions, and it was not till about 1860 that electrical registration was applied for obviating this. The instruments in use at the present time (see Figs. 25-27) vary fundamentally in the type of wheel, which in some, such as the Haskell, has screw-shaped blades, and in others, such as the Price, cups like those of the anemometer. Important types are frequently made in different sizes,- small ones, to be rigidly attached to jointed metal rods and designed for use in channels of moderate depth; and larger ones, to be suspended by flexible cord or cable and suited to the largest rivers. Nearly all forms are arranged for electrical registration;

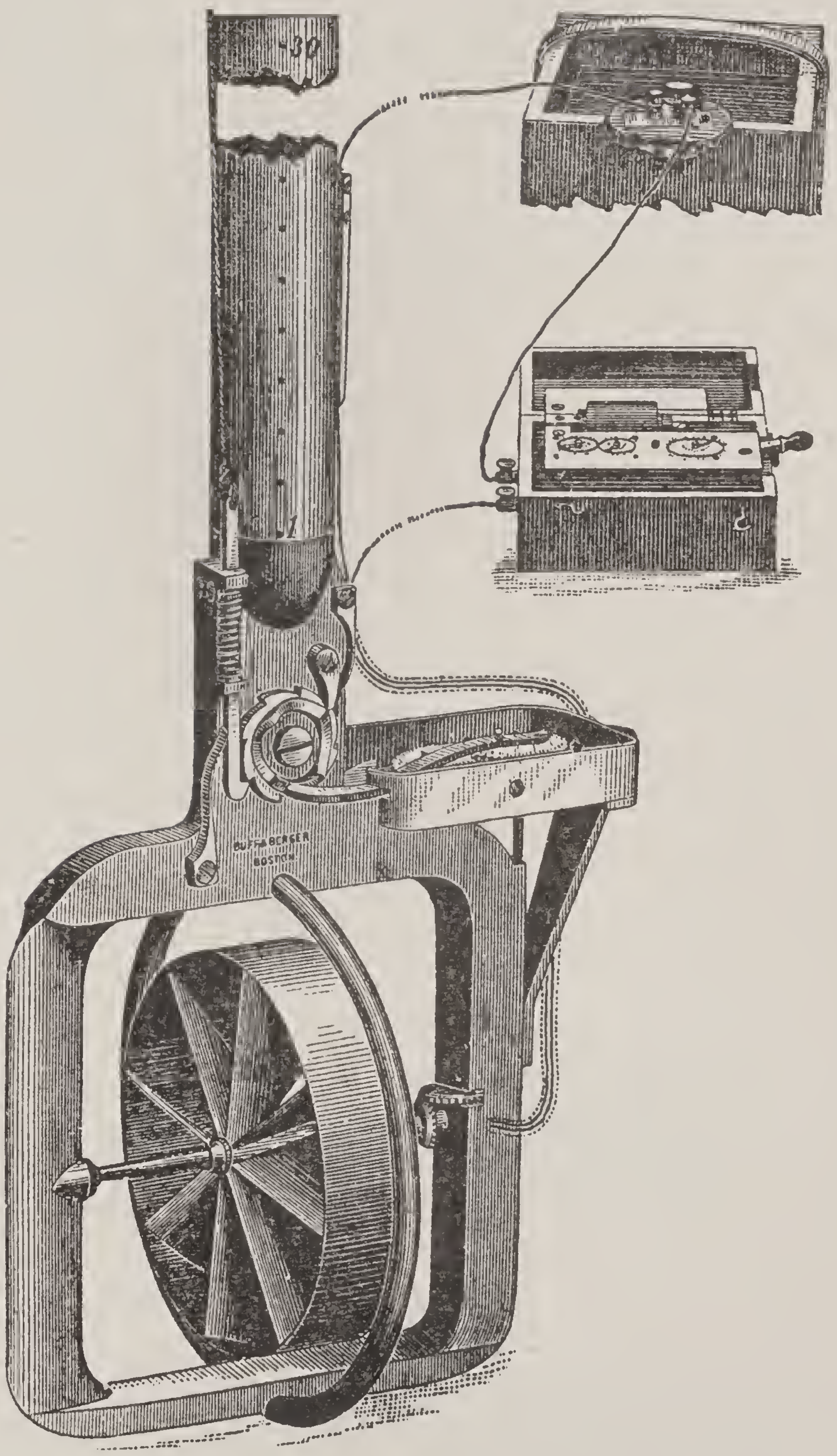


Fig. 25. Fteley Current Meter.

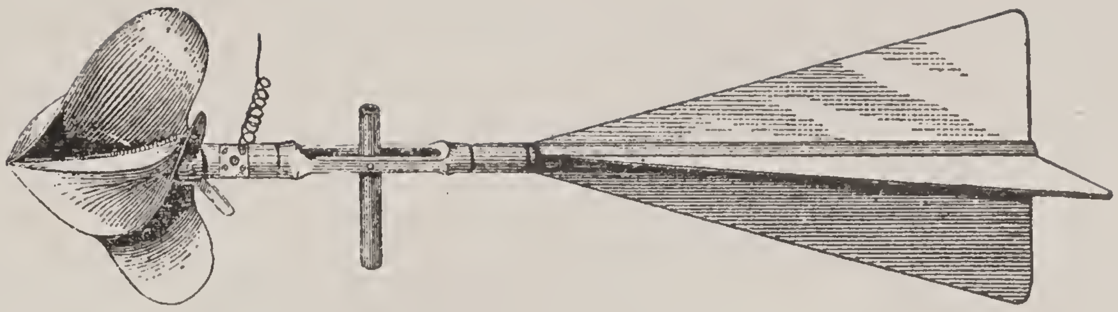


Fig. 26. Haskell Current Meter.

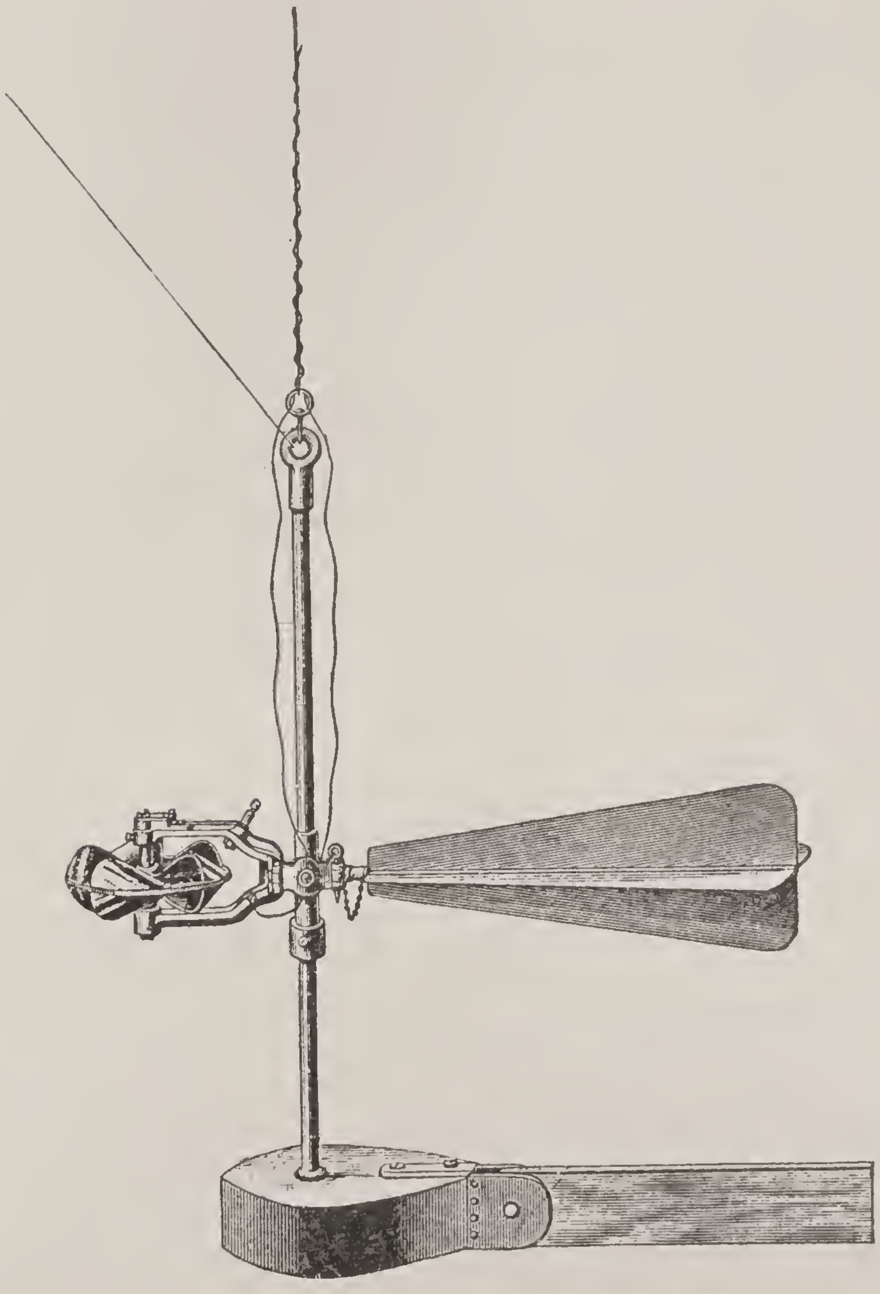


Fig. 27. Price Current Meter.

but some, such as the Fteley, provide also mechanical registration, through gearing; while one style, the "Price acoustic", announces by sound only the completion of each ten revolutions.

The relative value for general use of the screw or propeller type and of the anemometer type appears not to be clearly settled. Each is capable of doing excellent work. In the former the wheel axis is horizontal and the bearing surface greater than in the latter, in which the axis is usually vertical, with the wheel revolving in a horizontal plane. The friction is relatively greater, therefore, in the propeller type than in the anemometer, and the wheel likely to be less sensitive to sluggish currents. The Price wheel, which is of the anemometer class, is supported upon a small pintle, and the construction is such as thoroughly to exclude water and grit from the bearing surfaces. The bearing friction is, therefore, not only small, but tolerably constant also, even in silt-bearing water.

Both forms show noticeable error in registration when tipped at an angle to the direction of the current, as is likely to happen in working from an unstable support, such as a boat; the Price, for example, tending to over-register, and the Haskell (which is of the propeller type) to under-register, though to a less degree. The propeller wheel of the Haskell and the Hajós (Hungarian) types present a somewhat conical shape to the current, the resulting tendency of which is to throw off leaves and weeds which strike the wheel,- an advantage in streams in which these are running. The anemometer style usually has a longer pitch than the propeller, that is to say, it turns less rapidly for a given velocity of current, which is advantageous in measuring high velocities, for which it is difficult or impossible to count the revolutions mentally To the Price meter has even been added a special device by which only each fifth revolution is indicated, for use in rapid currents.

The Fteley meter is a small instrument having helicoidal vanes and used only upon a rod. It is of fine workmanship, sensitive, and has no superior for use in clear water of moderate depth; it is somewhat easily clogged by leaves and grass, however, and its bearings do not exclude silt or sawdust, so that it is not suited to streams carrying much matter in suspension.

The "Colorado" meter, designed especially for use in irrigation ditches, is attached to a rod, has a horizontal wheel, with cups, and is arranged to admit of its approaching very close to the stream bed.

The Price acoustic meter is also to be used with a rod. It has a small wheel with six cups, and at every ten revolutions of the shaft a little hammer is tripped and, striking a diaphragm, causes a clicking sound, which is transmitted through

the rod to the observer's ear. The gurgling of the water, if deep, and other noises often obscure the clicks, however, so that they cannot be distinguished; otherwise the instrument is excellent.

A meter fixed to a rod is completely under the control of the observer as to position in the water, but a suspended meter has to be provided with a tail to make it head into the current; and to prevent its being carried out of a vertical by the current pressure it must be more or less heavily weighted (ordinarily by from 10 to 60 pounds). The torpedo-shaped lead weight now adopted, suspended a little below the meter, is also provided with a tail to aid it in heading up stream. Even with a heavy weight it is often impracticable to keep the meter in the vertical in deep and swift currents, and a guy line from a transverse cable a hundred feet, more or less, up stream may then be needed.

In studying tidal currents it is sometimes desired to learn not only the speed of the current at chosen depths below the surface, but also the direction. For this purpose a direction attachment is made for the larger Haskell meters, consisting of a closed metal chamber inserted in the line of the meter axis, between the wheel and the tail, filled with oil, and in which is a compass needle, free to swing into the magnetic meridian. There is an electrical connection to a repeater box above water containing a compass, over the face of which a pointer may be slowly revolved until it has the same direction as the meter and current, when it stops and the compass bearing may be read. Direction apparatus devised by Leviavsky has been applied to studying the direction of sub-surface currents in hydraulic experiments in Russian rivers (Eng. News, Sept. 1, 1904).

Various methods are in use for ascertaining the number of meter revolutions in a velocity observation. That of the "acoustic" meter has already been mentioned. When operated without an electric current, the Fteley meter registers through gearing on a train of dials. Automatic electrical registering devices are supplied by the instrument makers, but a much simpler and far less expensive arrangement is a small buzzer, telephone receiver, or telegraph sounder; it is then necessary to count mentally the number of buzzes or taps, but the attention thus required is a safeguard against otherwise unperceived faults in the working of the equipment as a whole.

The Hydrographic Department of Hungary employs a very complete registering apparatus (described in Annales des Ponts et Chaussees, 1898,- III, IV), comprising an endless paper band which is automatically unrolled from its spool at a rate proportional to the descent of the meter through the water, and upon which is indicated, through the action of a specially-devised chronograph, each entire revolution of the meter wheel, each twentieth, or each hundredth, according to the velocity,

as well as each half second and each hundredth of a second.

The indication, by sound or otherwise, of the completion of successive revolutions of the meter wheel, is effected by the alternate making and breaking of an electric circuit. The details vary with the type of instrument, but the principle is perhaps sufficiently illustrated in Fig. 28, showing a section

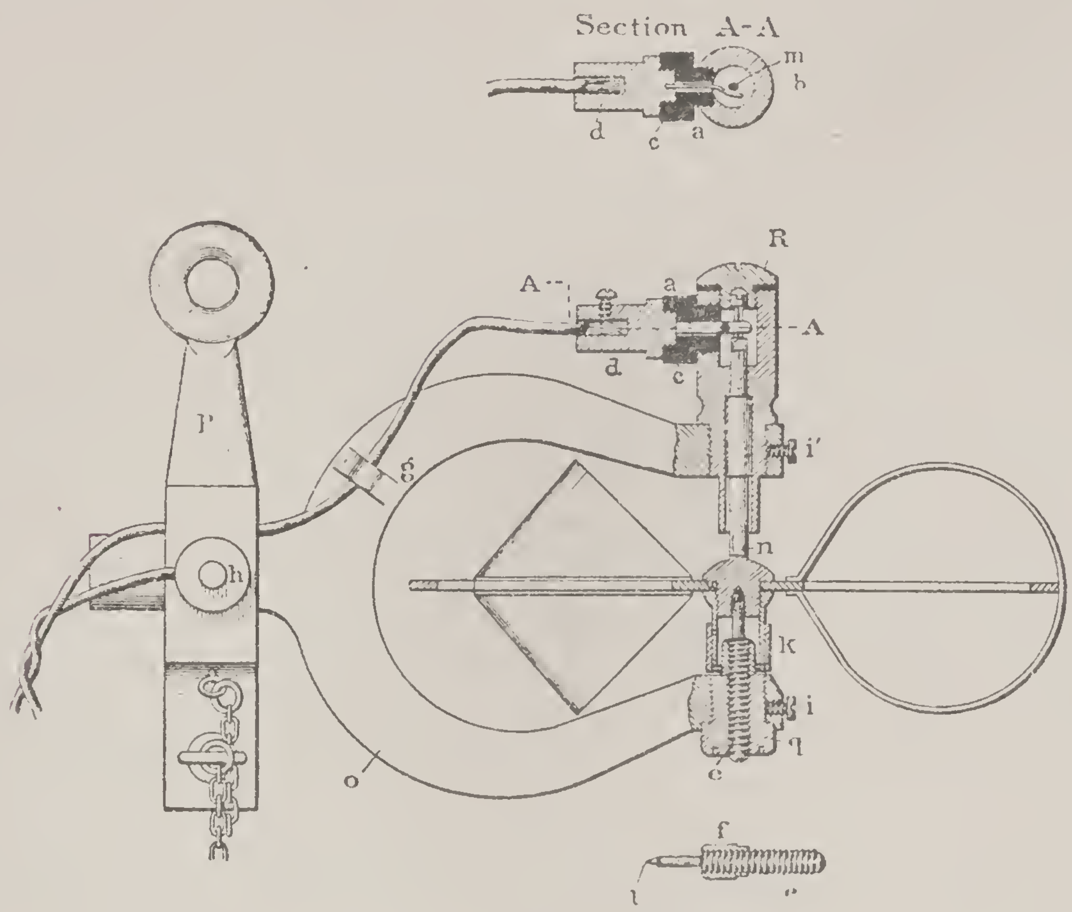


Fig. 28

of a Price meter. This cut is taken from U. S. G. S. Water Supply and Irrigation Paper, No. 94, in which are also given very complete directions for taking apart the meter head and replacing worn or injured parts. By means of flexible, insulated copper wires, which may serve at the same time for suspending a lightly-weighted meter in the water, the instrument is put into circuit with the recording device and battery above water. A switch is also introduced in the case of an automatic register, being thrown in at the beginning of an observation, and off at the end. For a buzzer or telephone receiver a small dry cell supplies all the current that is required. One wire of the circuit is connected to the metal of the meter at any convenient point (h in the figure); the other is in this case secured at the binding-post, d, whence there is connection via a delicate platinum spring, a, through the center of the hard rubber insulation, c, into the contact chamber surrounding the upper part of the meter shaft. This part of the shaft has an oval-shaped eccentric, m, which touches the spring in revolving and completes the circuit once in each revolution.

51. Rating the Meter. A current meter observation always consists in counting, or reading from dials, the number of wheel revolutions in a corresponding time interval, which may or may not have been decided in advance, the time being taken preferably with a stop watch. The average revolutions per second may now be computed, and are then to be translated into velocity of the water in feet per second. The relation involves the "pitch" of the wheel, or length of stream threads passing it for one revolution. With propeller wheels having accurately made helicoidal vanes, such as the Fteley, the pitch may be found with considerable closeness by measuring appropriate dimensions of the wheel, but in general each current meter is experimentally "rated" from time to time.

Almost universally this is done by moving the meter at as nearly uniform a rate as practicable over a measured distance, say from 50 to 300 feet ordinarily, in still water, and noting the number of revolutions and the elapsed time. Runs are made at various speeds covering the whole range likely to occur in practice, and the results plotted on cross-section paper, with velocity in feet per second as one co-ordinate, and revolutions per second as the other. A smooth "rating curve" is then passed among the plotted points. From this curve a rating table is usually drawn off, and is to be adjusted by differences as explained in Art. 40 for discharge tables. It should give velocities in general to two decimal places, and low velocities to three places.

Among the more noticeable methods of rating are:-

(a) Suspending the meter from a light truck moving on a track above and either over or at the side of the water. Devices may be added for automatically starting and stopping the watch and counter, and for producing uniform motion of the truck, although this is usually propelled by pushing.

(b) Suspending the meter from the bow of a boat, and propelling the boat by motor, cable or oars, over a measured course, although oars are inferior for this purpose.

(c) Suspending the meter from the extremity of a horizontal arm describing a circle about a pivot, the meter thus passing through a circumference of known length.

(d) Suspending the meter from a sled and moving it through a long slit in the ice on the surface of a pond.

With the special form of chronograph used by the Hydrographic Department of Hungary, whereby the relation between speed of revolution and speed of advance may be determined for successive small distances, even for those corresponding to but a single revolution of the meter wheel, it is claimed that expensive devices for moving the meter with uniform speed

through the water are unnecessary, pushing by hand being sufficient; and that by varying the speed at intervals during each run a very few trips over the measured course suffice, in place of the numerous trips, ranging from 25 to over 100, which are usual under the ordinary rating methods.

In rating, the meter is commonly placed 2 feet or more below the water surface (for high velocities it should perhaps be still deeper), and if suspended from a boat must be well away from the disturbance caused by the latter,- say at least 5 feet beyond the bow of an ordinary row-boat. Although with a truck an approximate rating of a small meter might be obtained in a trough a few square feet in cross-section, for an accurate rating a much larger section seems necessary, and a canal lock or reservoir is often made use of.

To guard against error from any small velocity that may exist in nominally still water, the runs should be made in pairs, that is, alternately in each direction at substantially the same velocity, and the mean for the pair used in fixing the rating curve.

The rating of a meter may become seriously altered by injury to cups or vanes, or by change in bearing friction through wear or change of contact apparatus, and possibly in minor degree by change in lubrication. Experience seems to indicate, however, that if the only change be in bearing friction all plotted ratings of the same meter should give parallel curves. A rough method often employed for testing the condition of the meter is to whirl the wheel in quiet air by a quick movement of the hand, and note the time required for coming to rest; if the time varies materially from that found immediately after rating, a change in rating is presumed.

A suitable stretch of still water is not always available, in which case resort may be had to the inferior method of rating in a current, provided it be steady and, preferably, slow. Let us suppose the meter suspended from a boat, and runs made in pairs, each pair comprising a run down stream and one up stream, both as nearly as practicable at the same absolute speed.

Let V_a and V_a' = observed absolute velocity of boat and meter in respective runs of any pair,
V_r and V_r' = their corresponding relative velocity with respect to the water,
V_w = corresponding absolute velocity of water, supposed to be constant.

Then, with the current, $V_r = V_a - V_w$,

and against current, $V_r' = V_a' + V_w$.

$$\therefore \frac{V_r + V_r'}{2} = \frac{V_a + V_a'}{2},$$ and we may plot for each pair the mean value thus found.

Moving water is sometimes employed also to give a partial check upon meter ratings by comparing the observed speed of floats with that indicated by meter. A more comprehensive check is sometimes practicable where the entire stream discharge may be determined simultaneously by meter and by a standard weir. Subject to whatever error may exist in the weir measurement, a coefficient may thus be found for correcting a meter rating made in still water and adapting it to the conditions of moving water (see Cornell experiments by E. C. Murphy, U.S.G.S. Water Supply and Irrigation Paper, No. 95, pp. 80-82).

That the conditions are not precisely the same in rating a meter in still water and in using it to gage running water has been pointed out by Stearns, who calls attention to the following differences (Trans. Amer. Soc. Civ. Engrs., Vol. 12, 1883):-

(a) The velocity of the meter through still water may be substantially uniform, but the velocity of running water in contact with the meter varies through pulsations, and in integration varies with the changing position of the meter.

(b) In rating, the direction of motion may practically coincide with the axis of a screw meter, or the plane of revolution of a cup meter; but in a stream the water may strike the meter obliquely, because of eddies or cross-currents, or faulty holding of the meter; and in integration this obliquity is sure to occur, since the effective direction of the water with respect to the meter is that of the resultant of the motion of the current and the reverse of the motion of the meter.

(c) In rating, the forward motion is the same for all parts of the meter, while in a running stream the water may vary considerably in velocity within the diameter of the meter wheel. In Stearns' experiments a difference as great as 10% existed within a distance equal to the diameter of the Fteley meter wheel, near the bottom and sides of the Sudbury conduit.

Assuming now, for simplicity, the case of a propeller meter with helicoidal vanes, it will be seen that if there were no solid or fluid friction, and no disturbance of the water by the meter frame and vane edges, the wheel, considered as a screw, would make the same number of revolutions in traversing a given distance through still water, whatever might be its forward speed through the water; and the revolutions per second would be directly in proportion to the forward speed. A line such as a-b (Fig. 29) would represent the first-mentioned relation, and o-d (Fig. 30) the second. In fact, however, there is a bearing friction such that some noticeable velocity,

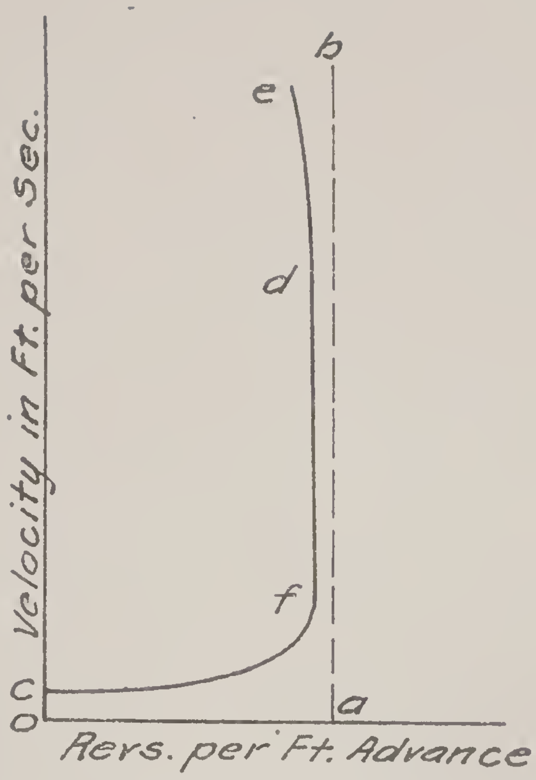


Fig.29

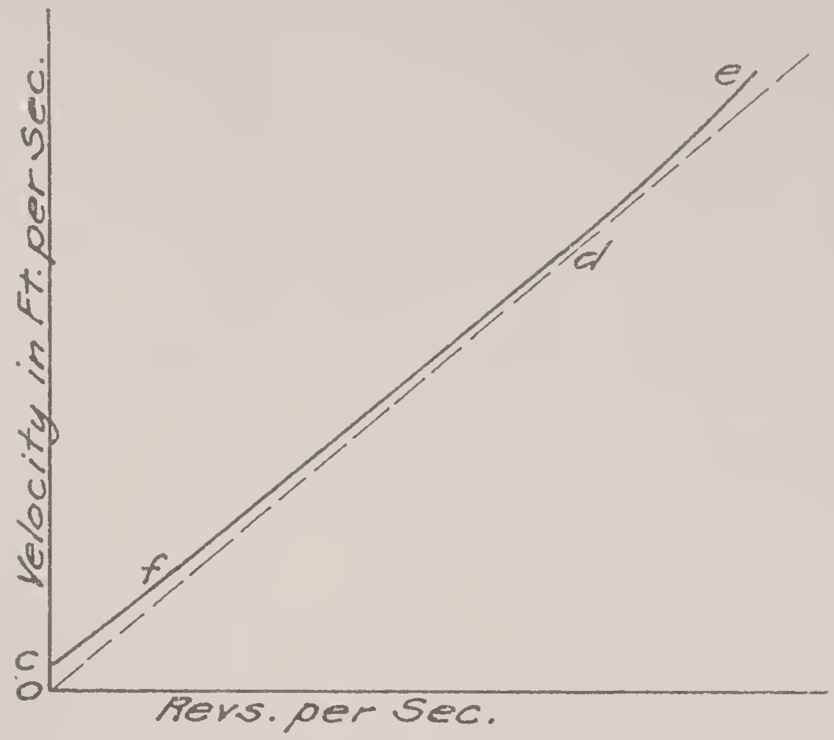


Fig.30

varying from say 0.5 foot per second down to less than 0.1 foot per second with different meters, is required to start the wheel revolving. This is represented by the distance o-c.

The resisting force due to bearing friction does not materially increase, however, as the velocity through the water increases, but, on the other hand, relatively diminishes as compared with the force developed by the latter, the result being that its effect in reducing the theoretic number of revolutions rapidly decreases as the speed increases, and at moderate velocities becomes small and approximately constant. In consequence the rating line as a whole is concave toward the axis of velocities, the curvature being most pronounced over the distance c-f, that is, for velocities less than from one-half foot to two feet per second, according to the particular meter or type of meter used. From f on, say to d, or to above the velocities usually encountered, the line is either practically straight, or has a very flat curvature. There is some resistance to turning due to the fluid friction of the water in contact with the whirling wheel, this friction presumably increasing approximately as the square of the velocity and, according to Hajos, becoming sensible above velocities of say 10 feet per second, and tending to cause increased concavity, as at d-e.

A slight change in bearing friction has so large relative effect upon revolutions, at very low velocities, and the difference between discharge as measured by weir and as measured by current meter has been found so large at low velocities, that it is considered safer not to use the meter for gagings in velocities below about one-half foot per second.

For some meters the rating line as a whole shows so little curvature that it is drawn as a straight line among the plotted points representing the results of the various observations. If the points indicate noticeable departure from a straight line, however, it is well to draw the rating line accordingly. This line is usually located by eye among the plotted points, being drawn by aid of a straight-edge, sprung to a flat curve if necessary; but not infrequently the method of least squares is applied to finding the equation of the line, assuming the typical form $y = ax + b$ for straight lines, and $y = a + bx + cx^2$ for curved lines, and Murphy considers that this method should be used if high accuracy is sought from meter measurements at low velocities. It is perhaps fair to claim that, in a first-class rating, plotted points should seldom vary more than 1% from a smooth rating curve.

It is thought that the results of a rating are best shown by a curve constructed as in Fig. 29, and that values can be read from it more accurately than from one constructed as in Fig. 30; but, for direct use in connection with gagings, either a curve of the latter form, based upon revolutions per second, or an equivalent rating table, is required.

52. Details of Current-Meter Measurements. The observations may be made either by the method of point measurements, as at the assumed depth of mean velocity, or at mid-depth, or at a series of points in a vertical; or by the method of integrated measurements, as previously described. Point measurements in a regular current appear to reproduce the conditions of rating more nearly than do integrated measurements, and therefore have some theoretic advantage. The practical management of the meter will be mainly governed by the depth and velocity of the current, but the following plans may be noticed:-

(a) Operating a rod meter by hand while wading, as in shallow streams.

(b) Operating from a bridge, using rod meter or suspended meter according to depth of water and height of bridge.

(c) Operating from a row-boat, either anchored or secured to a cross line. The meter may be suspended to advantage from the end of a projecting boom, or lowered and raised upon a standing line secured to a heavy weight resting on the stream bed.

(d) Operating from a launch or catamaran, anchored or not according to circumstances. This is necessary in large and swift rivers. On the Mississippi the meter is often suspended by a cable from a boom projecting obliquely from the stern of the launch, and is kept vertical by being weighted, and by means of a guy line running from the meter to the end of a boom in the bow of the launch, or to a sheave on a bow anchor and thence to the launch.

(e) A cable is suspended over the stream from bank to bank, and the meter is operated from a sling or chair hung from the cable. This method has been used in very swift rivers, where operations would otherwise have been dangerous.

(f) A cable is suspended over the stream, and the meter is run out from the shore, being hung from a carrier and guyed from a second cable farther up stream. Both these methods have been used by the U. S. Geological Survey (see 11th Annual Report, 1889-'90, Part 2, p.15).

The length of an observation should be fixed with reference to the time allowable for, and the general accuracy aimed at in, the gaging as a whole; and in particular with reference to obtaining a mean of the pulsations of the current and to keeping relatively small the errors due to vertical motion of the meter in integrating, inaccurate timing, or failure to record partial revolutions.

In gagings for the Mississippi River Commission, point measurements appear usually to have been continued for from 2 to 10 minutes, averaging perhaps from 3 to 5 minutes.

In integration, uniformity of vertical motion is important. The observer will be aided in securing this by having an assistant announce regular time intervals corresponding to a given rise or descent of the meter, although with practice he may secure a sufficiently uniform motion without such aid. The rate of vertical motion must also be slow enough to prevent its affecting materially the registration of the meter; it must therefore be small as compared with the horizontal velocity of the current, Stearns' experiments indicating that for the Fteley meter 1 to 20 is a safe ratio.

In integration, the elapsed time must be taken on completing the vertical, and also the number of revolutions, which for nearly all meters can be observed to the nearest integral only; while in a point measurement we may attempt to observe either the revolutions in an integral number of seconds, or the number of seconds for an integral number of revolutions. Since the number of revolutions n per second equals the total number N during the observation, divided by the number of seconds t, it will be seen that when N is large compared with t, as with a short-pitch meter in a rapid current, it is the more accurate to base the observation upon an integral number of seconds; but, when N is small compared with t, to base the observation upon an integral number of revolutions.

There is risk of error if the meter be used near the stream surface, experiments by Murphy with the small Price meter, largely employed by the U. S. Geological Survey, having shown the error to be material if the meter be less than 6 inches below the surface in currents of more than 1½ foot per second velocity.

Accurate results are not to be expected from the meter in sluggish currents, and experience has led the U. S. Geological Survey not to regard as suitable for a gaging station a cross-section showing less than one-half foot per second velocity in more than 15% of the area.

Field notes should record:- Names of observers; name of stream; locality of gaging, with sketch of site; date of gaging; time of beginning and of ending gaging; name and number of meter; gage height at suitable intervals for determining mean value and for adjusting soundings; reference of zero of gage to permanent bench mark; test of watch for error; distance out from reference line to station; water depth; depth of observation; reading of watch at beginning and end of observation; revolutions of meter wheel. If computations are to be made in the field, there will also be added, in suitable columns,- revolutions per second; velocity in feet per second; width, mean depth, and area of vertical strips of cross-section; discharge past strips; and area, mean velocity and discharge for entire section.

53. The Pitot Tube. In its simplest form,- a vertical tube, with a right-angled bend at the bottom pointing up stream - this device was invented by Pitot in 1730. The impact of the current against the opening causes water to rise to, and in a steady current to remain at, a height above the free surface of the stream approximately equal to the velocity head at the orifice. That is, referring to Fig. 31,

$h = \frac{v^2}{2g}$, nearly,- whence $v = \sqrt{2gh}$. If the elbow be pointed down stream, some suction is likely to result and the column within the tube to sink even below the free surface outside.

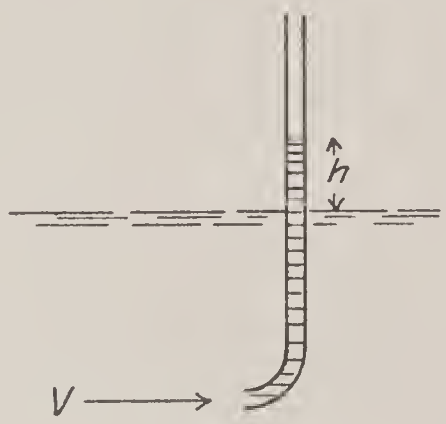


Fig. 31

The value of h generally being small, and the free surface of a stream more or less disturbed, there are manifestly difficulties in the way of using this simple form for stream gaging. About 1865 Darcy and Bazin developed the instrument farther by using two orifices,- one pointing up stream and made very small so as to avoid disturbing the stream lines and to lessen the oscillations within the tube, and the other, also small, pointing at right angles to the current. The principle will appear from Fig. 32, where it will be seen that h' measures simply pressure head, while h" measures pressure head plus velocity head, and as before

$h = \frac{v^2}{2g}$. But it is now possible, by

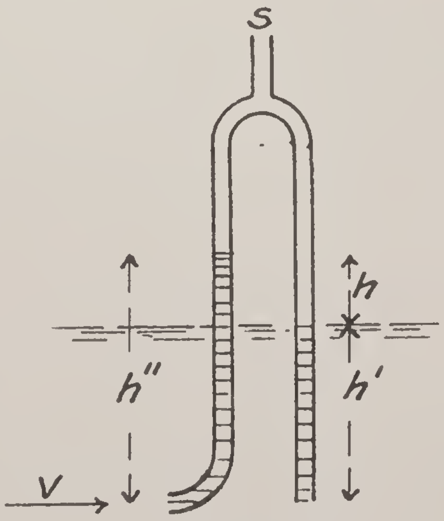


Fig. 32

applying a moderate suction with the mouth at s, to raise both columns to a convenient elevation for reading, without changing their difference. The perfected instrument has a scale for reading the water columns, and stop-cocks for holding the vacuum and the water columns in taking a reading.

The value of h is so small that but little accuracy is to be expected at velocities below about one foot per second from vertical water columns. The value of h may be exaggerated, however, by inclining the columns, or by using oil in the upper part of the tubes in place of air and thus forming a differential oil gage (see paper by W. B. Gregory in Trans. Amer. Soc. Mechan. Engrs., Vol. 25). The Pitot tube is unsuited to turbulent water, and has the general disadvantage of indicating velocity at a single instant only; but in smooth, swift currents, especially in small channels and to depths not exceeding say 5 feet, it may often serve a very useful purpose.

Ritter proposed an ingenious special form for use in measuring surface velocities in flood gagings when working from a bridge, the part containing the tips being weighted and provided with a vane to insure heading into the current, while flexible tubes transmitted the pressure to the observer's manometer on the bridge. A peculiar feature was the introduction of coiled capillary tubes between the orifices and the flexible transmission tubes, so as to exclude water from the latter, compressed air thus becoming the medium for transmitting the pressure to the manometer (Annales des Ponts et Chaussees, 1886, 2d semestre, pp. 697 et seq.).

Practically, $v = c\sqrt{2gh}$. For an impact tube alone, if given a small, thin-edged orifice, c would doubtless be closely unity; but the suction produced by the second, or pressure tip, usually increases the observed h, so that c may fall materially below unity.

Experience indicates that care must be given to the rating of each instrument of the type of the Pitot tube, especially where two orifices and transmission tubes are employed, and that the rating should be performed in conditions as nearly as possible duplicating those to be met in the actual gaging. Thus, experiments by E. C. Murphy (Eng. News, Aug. 12, 1909) showed the column difference to vary considerably, for the same velocity, as the orifice tip was moved farther and farther from the channel bed; and showed the mean velocity in an experimental trough, as obtained from a gaging with a tube rated in still water, to vary from 3 to 11% from the value obtained from the discharge independently measured.

Three principal methods have been used in rating such instruments:-

(a) Comparing the readings of the tube with the velocities shown by floats run over a definite course.

(b) Securing the tube to a boat or truck and drawing it through still water at different observed speeds.

(c) Holding the tip at many points in the cross-section of a channel, the discharge of which could be simultaneously measured by other means, such as a weir.

Darcy used methods (a) and (c), adopting the mean of the values thus found.

54. Measurement of Discharge by Chemical Means. In an article by Charles E. Stromeyer in Min. Proc. Instn. Civ. Engrs., 1905, Vol. 160, pp. 349 et seq., he describes a chemical method of gaging flow which he has used in measuring the supply of feed water and circulating water for engines, and with which he has also experimented in small rivers, sewer outfalls, etc. In the experiments for which results are given the error ranged variously from between 1 and 2% to between 7 and 8%. Quoting from the above article:- "In gaging by the chemical method, a fairly concentrated solution of some chemical for which very sensitive reagents are known is discharged, at a uniform and accurately known rate, into the stream under observation, and analyses are made of the water of the stream before and after the addition of the chemical. The ratio of the percentage of chemical in the concentrated solution to the percentage added to the water is obviously the same as the ratio of the volume of flow of the water per second to the volume of flow of the solution per second. Various salts can be used; in general, acids or alkalis would be employed with brackish or sea water, and chlorides with hard water."

Let x = weight of flow of stream per second (not materially altered by amount of chemical solution added),

W_c = weight of flow of chemical solution per second,

r_1, r_2, r_3, = ratios of chemical in approaching stream, added solution, and departing stream, respectively,

Then $x\,(r_3 - r_1) = W_c\, r_2$

$$\therefore x = \frac{r_2 W_c}{r_3 - r_1}.$$

The conditions necessary to success by this method are that the flow be sufficiently swift and agitated to ensure thorough mixing of the chemical with the stream in a moderate distance, and that between the two sampling stations there be no marked variation in impurities other than that due to the chemical purposely introduced. Colorimetric analysis has been suggested for use, by which a suitable coloring matter, eosine for example, should be added, and the degrees of dilution determined by comparison of samples with standards of known dilution, but experiments by Belcher and Colson, M.I.T., 1907, indicated that

reasonable accuracy in estimating the dilution could be obtained only by adding an excessive amount of coloring solution; they concluded that for streams of any considerable volume gravimetric analysis would be necessary for accurate results, and that more chemical skill would be required than is ordinarily possessed by the engineer.

In exceptional cases a suitable coloring solution has been injected into the stream flowing in a closed conduit, serving as a substitute for floats, the color enabling the eye to note the time of arrival at some manhole or other opening, and thus affording means of approximately determining the mean velocity for the whole cross-section.

55. Measurement of Relative Discharge by Use of Thermometer. For the quick approximate measurement of relative discharge, especially of small mountain streams whose turbulence would forbid the use of meters or floats and for which the establishment of weirs is for any reason inadvisable, the thermometer may sometimes be employed. The method was described at length by C. Ritter in a paper entitled "Emploi du Thermometre dans le Jaugeage des Petits Cours d'Eau" (Annales des Ponts et Chaussees, 1884, pp. 323 et seq.).

It is to be applied at the junction of two streams whose temperatures are sensibly different, but uniform throughout the cross-section of each, as well as throughout the cross-section of the main stream below the junction. Three stations are therefore made use of,- one on each separate stream shortly above their junction, and one on the combined stream far enough below the junction to permit thorough mixing to have been effected. Ritter employed thermometers graduated to fifths of a degree Centigrade, and took the temperature of the water which he removed from the stream in a small wooden pail having no metal trimmings. He did not consider the operation good unless several successive attempts gave exactly the same results.

Let Q = volume per second flowing in one stream above junction.
Q' = " " " " other " "
t and t' = their respective temperatures,
T = common temperature below junction.

Then, by the law of mixtures, $\frac{Q}{Q'} = \frac{t - T}{T - t'}$.

If the absolute volume at any one of the three stations be known from other data, then not only the relative but also the absolute values of Q and Q' become known. Ritter cites two cases in which volumes of about 4 and about 40 cubic feet per second, respectively, were measured by this means with errors of about 2% and 6%, respectively.

56. Computation of Discharge by Means of Slope Formulae.

From the general formula $V = C\sqrt{R\,S}$ it appears that, if C, R and S can be accurately learned, V and hence Q can be determined without a direct instrumental gaging. For this purpose a length l must be chosen, great enough if practicable to keep within satisfactory limits the relative error in determining S, and in which the slope and cross-section are tolerably uniform; the fall h in water surface for this distance l (which, strictly speaking, is to be measured along the slope) is to be found from simultaneous readings on gages of known elevation placed at the extremities of the reach, whence $S = \frac{h}{l}$. The average hydraulic mean depth R for the reach l must also be found, and the proper value assigned to C.

The practical difficulties of measuring S, which is usually small, with sufficient accuracy and refinement, and of choosing the correct value for C, are so great that the resulting value of Q is often liable to large error, and the method does not commend itself except when direct gagings are out of the question. It is sometimes applicable, however, as a last resort in determining what was the approximate discharge of a river in a flood that has passed, marks along the banks giving the means of roughly ascertaining the slope.

57. Accuracy of Stream Gagings. An absolute test of the accuracy of a stream gaging must generally involve a simultaneous independent measurement of the volume flowing, made by some method known to be of superior accuracy. This is seldom practicable on a large scale, and one must resort to such evidence as is afforded by comparison of gagings made at the same time with different meters, or with meters and floats. Many such comparisons have been instituted, to a few of which reference will be made.

In 1874, extensive gagings of the Connecticut river at Thompsonville, Conn., were made by Gen. Theodore G. Ellis, using both double floats and current meters. The observations were scattered over a period of about four months, during which the discharge ranged, in round numbers, from 4,000 to 63,000 cubic feet per second. One of the most important conclusions reached from a study of the gagings was stated to be, "That both the floats and meters give concordant results, showing that each is reliable when carefully and intelligently employed, and that each method has certain advantages over the other under peculiar circumstances" (Annual Report, Chief of Engrs., U.S.A., 1878, App. B, 14, p. 11).

In the discussion of William Starling's paper on "The Discharge of the Mississippi River", Arthur Hider cites ten gagings made in pairs on the lower Mississippi in 1892, each pair comprising a current meter gaging made in the forenoon

and a double-float gaging made in the afternoon of the same day, the discharge ranging for the entire period, in round numbers, from 1,000,000 to 1,500,000 cubic feet per second. Both sets of observations in each pair were taken at 6/10 depth. Usually, though not always, the meter gave a less discharge than the floats, the differences ranging as a whole from about 4% downward nearly to zero (Trans. Amer. Soc. Civil Engrs., Vol. 35, p. 330).

As the result of experiments made by James B. Francis to test the degree of uniformity attainable in gagings with tube floats in rectangular canals, he concluded that, "We must infer from these seven experiments that any single measurement is liable to be erroneous to the amount of 1%, or perhaps rather more; and in any two experiments the errors may be in opposite directions, in which case they may vary from each other 2%, or rather more (Lowell Hydraulic Experiments, p. 200).

The writer's experience with students making for the first time tube float measurements in rectangular canals in which the discharge varies say from 700 to 1500 cubic feet per second, simultaneous gagings being made by independent parties, is that the difference in discharge for the two gagings of a pair seldom exceeds 2%, and averages not more than 1½%.

Very elaborate experiments were made in 1900 by E. C. Murphy, of the U. S. Geological Survey, in the Cornell University flume, for the purpose of testing the accuracy to be obtained by current meters used under different methods. The discharge ranged, roughly, from 200 to 235 cubic feet per second. In no case out of 50 comparative measurements did a meter discharge differ from a corresponding weir discharge by as much as 5%; nor did two simultaneous meter discharges vary from each other by as much as 5%. The tests were thought to show that, under favorable conditions, discharge can be measured with a Small Price meter (one of the types used in the experiments) with an error of not more than 1%, although in ordinary river sections so high a degree of accuracy could not be expected (Trans. Amer. Soc. Civil Engrs., Vol. 47, pp. 370 et seq.).

E. E. Haskell, an authority upon stream gagings, in discussing the paper above referred to, expressed the opinion that "The Niagara river, and similar streams of comparatively permanent regimen, can be measured, with residuals not exceeding 2% of their flow"; and that "The discharge of the Lower Mississippi River, with its ever-changing conditions, can be measured, with residuals not exceeding 5% of its flow."

It must not be forgotten, however, that if such accuracy is to be obtained with the current meter, then, as stated by Haskell, the instrument "must be of good design, well made, well rated, well cared for, and used by a careful, painstaking observer."

58. Time Required for Gagings. So long as the rate of flow of a stream remains constant, no limit need be set to the duration of a gaging, except that resulting from considerations of economy. But, in general, streams fluctuate so rapidly in discharge that it is desirable to finish observations for velocity in as short a time as allowable, in order to avoid error from a change occurring in velocities throughout the cross-section during the progress of the measurement. The time allowed, therefore, for a current-meter gaging, even of a large river, such as the Niagara or lower Mississippi, seldom exceeds from 2 to 4 hours. A very complete gaging with rod or tube floats can be made in an hour or less in a canal of 50 feet width, and a meter measurement by the method of vertical integration can be accomplished in even less time; a point measurement is likely to require more time than one by integration, but dependent upon the number of observations in each vertical.

59. Gagings in Ice-Covered Channels. It is often required to measure the discharge of rivers and canals when frozen over during the winter season. The ice covering produces a large surface friction, which greatly alters the distribution of velocities from that existing during open-water conditions, thus forbidding the use of the ordinary reduction coefficients. When the covering is continuous and smooth, fairly constant velocity relations are found to hold, upon which gagings may safely be based, and station rating curves may also properly be developed. But if a stream be covered with broken and tilted ice, or contain much needle ice, the velocity relations become too variable and uncertain for use.

Because of the greater friction and the resulting smaller velocity, the carrying capacity of a given wetted cross-section is lessened by surface freezing, and the discharge can be maintained only by an increase of section and consequently of gage height. After a thick ice sheet has been formed, however, it may sufficiently withstand pressure from beneath to permit of the stream flowing with moderate pressure head, and even with greater velocity than prevailed in the same cross-section when uncovered. A marked effect of the ice covering is seen in the vertical velocity curves, the surface velocity being so reduced and the concavity so increased that two points of mean velocity are found in each vertical (Fig. 33).

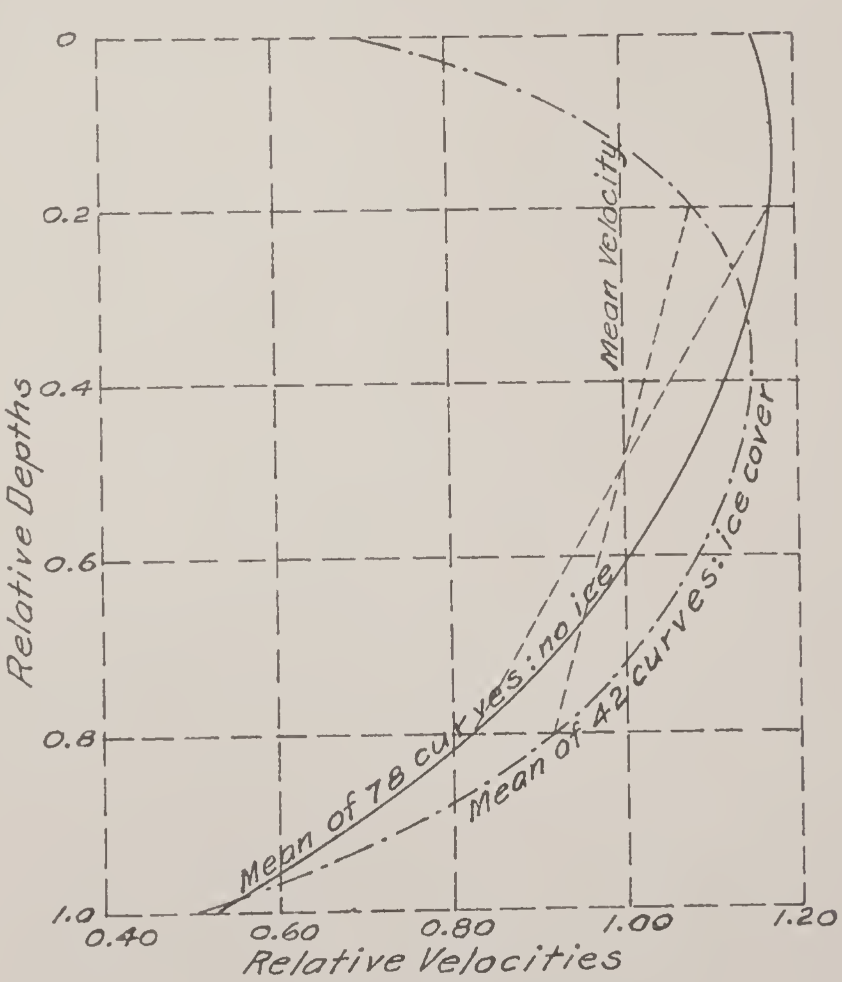


Fig. 33

Gagings of ice-covered streams can be made most accurately by observing velocities at sufficient points in each vertical to define the vertical velocity curves. Vertical integration may often be employed to advantage, although the formation of an ice coating on meter cable or rod is troublesome at low temperatures. Where enough vertical velocity curves have been determined to establish reliable reduction coefficients, subsequent observations may safely be made, on streams having a smooth ice cover, at one or two points only in each vertical, as in the case of open streams.

A study of about 350 vertical velocity curves, taken in various ice-covered streams and in different conditions, has shown the average position of the two threads of mean velocity to be very closely 1/10 and 7/10 depth, respectively, measured from bottom of ice (U.S.G.S. Water Supply and Irrigation Paper, No. 187, p. 79). There appears to be greater constancy, however, in velocity relations in the central part of the verticals. The reduction coefficient to be applied to the velocity at mid-depth, reckoned from bottom of ice, to give mean velocity past the entire vertical, averaged 0.88 for the curves above referred to, and varied but little at any single gaging station with change of stage. For two observations in a vertical, 2/10 and 8/10 depth below bottom of ice have been found to give excellent results, the mean of the velocities at those depths giving, on the average, very exactly the mean velocity past the entire vertical.

Comparatively few station-rating discharge curves for conditions of ice covering have thus far been constructed. They must evidently lie, on the whole, nearer the axis of gage heights than the corresponding curves for open water

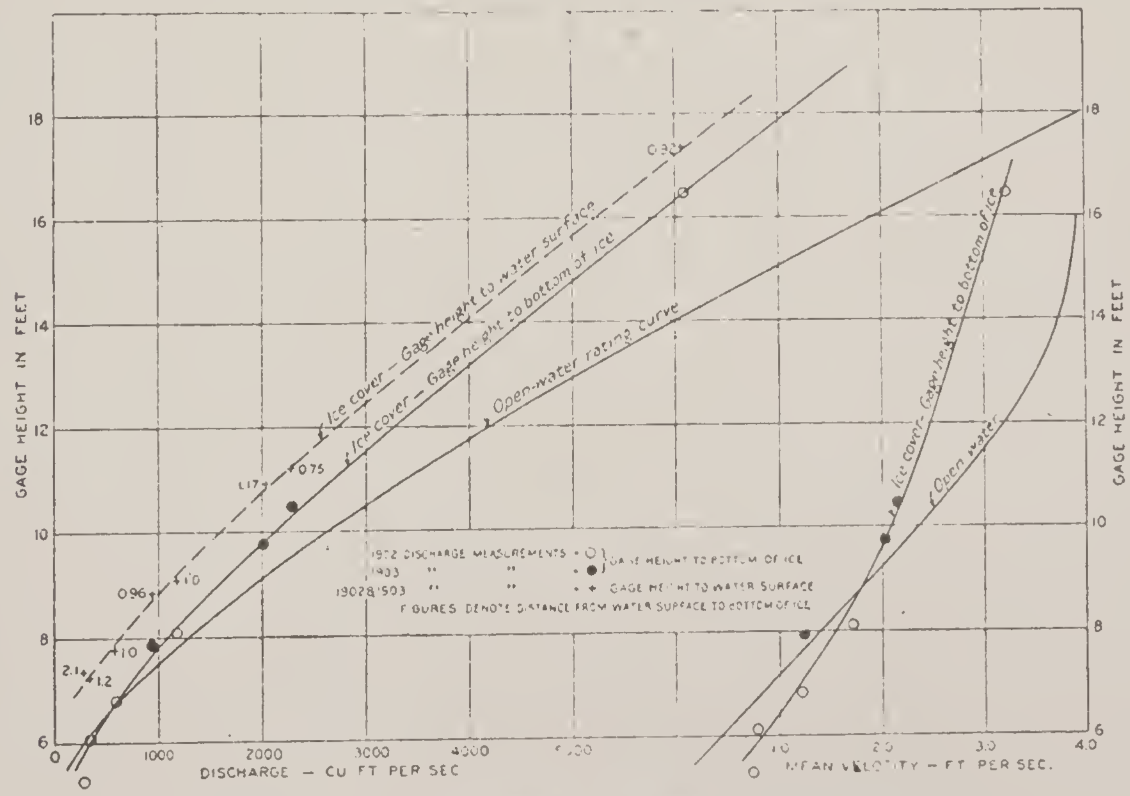


Fig. 34. Wallkill River at Newpaltz, N. Y.

conditions, and will assume different positions accordingly as gage heights are measured to the lower surface of the ice, or to the water surface in a hole cut in the ice (see Fig. 34, from U.S.G.S. Water Supply and Irrigation Paper, No. 187, p. 43). The effect upon such curves of varying thickness of ice, or varying roughness of surface, has not been well determined.